k-t PCA: Temporally Constrained *k-t* BLAST Reconstruction Using Principal Component Analysis

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INTRODUCTION: The *k-t* BLAST technique [1] and its parallel imaging counterpart *k-t* SENSE [1] have become widespread for reducing imaging speed in dynamic MRI. In its basic form *k-t* BLAST speeds up the data acquisition by undersampling *k*-space over time (referred to as *k-t* space). The resulting aliasing is resolved in the Fourier reciprocal *x-f* space (x = spatial position, f = temporal frequency) using an adaptive filter derived from a set of low-resolution training images. However, this filtering process tends to increase the reconstruction error or lower the achievable acceleration factor. This is problematic in applications exhibiting a broad range of temporal frequencies, such as real-time cardiac imaging. We show that *temporal basis functions* spanning the time-variation of the imaged object can be used to constrain the reconstruction such that the temporal resolution of *k-t* BLAST is improved. The temporal basis functions are calculated by subjecting the training data to principal component analysis (PCA). Accordingly, the presented technique is called *k-t* PCA.

THEORY: This section describes *k-t* PCA reconstruction for R-fold undersampling of *k-t* space on a sheared grid [1,2]. All steps of the reconstruction are described by the equations on the right. First, the low-resolution training data in *x-f* space are decomposed using PCA (Eq. [1]). This yields a set of temporal basis functions ($\mathbf{B}$) with corresponding spatial weightings ($\mathbf{w}_{train}$). As described by Eq. [2], the aliased *x-f* signal (ρ_{alias}) is given by the sum of four voxels in the true *x-f* object (ρ). Equation [3] states the fundamental assumption of *k-t* PCA, namely that the true *x-f* object is given by a linear combination of the temporal basis functions ($\mathbf{B}$) with unknown spatial weightings ($\mathbf{w}$). Now, the aliased signal can be expressed using $\mathbf{B}$ and $\mathbf{w}$ (Eq. [4]). Since the spatial weighting ($\mathbf{w}$) is temporally invariant, Eq. [4] must be true for all temporal frequencies (f_m) at a specific spatial position (x). By collecting in the vector $\boldsymbol{\rho}_{alias,x}$ the aliased signal of all N temporal frequencies at spatial position x (Eq. [5]), and by collecting in the vector $\mathbf{w}_x$ the unknown spatial weightings at the four aliasing (spatial) positions (Eq. [6]), it is possible to assemble a signal encoding matrix $\mathbf{E}$, such that $\boldsymbol{\rho}_{alias,x} = \mathbf{E}\mathbf{w}_x$ (Eq. [7]). The encoding matrix is constructed from copies of $\mathbf{B}$, which are shifted according to the sampling point spread function. Sensitivity encoding can be included in $\mathbf{E}$ if desired. The regularized least-squares solution to Eq. [7] is given by Eq. [8], where $\boldsymbol{\Theta}$ denotes the signal covariance matrix (i.e., an estimate of $\mathbf{w}_x$) calculated from the training data ($\mathbf{w}_{train}$). With PCA, most of the variation within the data of interest is usually modeled within the first few principal components, such that in practice the number of temporal basis functions can usually be reduced to 10-15. Importantly, if the number of temporal basis functions is sufficiently low (i.e., if number of temporal basis functions $\times R \leq N$), *k-t* PCA reconstruction is an overdetermined problem (i.e., it has more equations than unknowns) as opposed to *k-t* BLAST, which is always underdetermined. As a final remark, we note that if $\mathbf{B} = \mathbf{I}$ (i.e., identity), *k-t* PCA and *k-t* BLAST are mathematically equivalent.

$$1.\quad \rho_{train}(x_i, f_{m,i}) = \mathbf{B}(f_{m,i})\mathbf{w}_{train}(x_i)^T$$

$$2.\quad \rho_{alias}(x, f_m) = \sum_{i=1}^{R} \rho(x_i, f_{m,i})$$

$$3.\quad \rho(x_i, f_{m,i}) = \mathbf{B}(f_{m,i})\mathbf{w}(x_i)^T$$

$$4.\quad \rho_{alias}(x, f_m) = \sum_{i=1}^{R} \mathbf{B}(f_{m,i})\mathbf{w}(x_i)^T$$

$$5.\quad \boldsymbol{\rho}_{alias,x} \equiv \left[\rho_{alias}(x, f_1) \quad \rho_{alias}(x, f_2) \quad \cdots \quad \rho_{alias}(x, f_N)\right]^T$$

$$6.\quad \mathbf{w}_x \equiv \left[\mathbf{w}(x_1) \quad \mathbf{w}(x_2) \quad \cdots \quad \mathbf{w}(x_R)\right]^T$$

$$7.\quad \boldsymbol{\rho}_{alias,x} = \mathbf{E}\mathbf{w}_x$$

$$8.\quad \mathbf{w}_x = \boldsymbol{\Theta}\mathbf{E}^H(\mathbf{E}\boldsymbol{\Theta}\mathbf{E}^H + \lambda\mathbf{I})^{+}\boldsymbol{\rho}_{alias,x}$$

METHODS: To demonstrate the performance of *k-t* PCA, we used fully sampled real-time cardiac images acquired on a 3.0T MR system (Gyroscan Achieva, Philips Healthcare, Best, The Netherlands) equipped with a six-element cardiac receive coil. The images were acquired with a matrix size of 128x128 at a frame rate of approximately 10 frames per second. A total of 128 consecutive frames were acquired. By picking relevant phase encoding profiles from the fully *k*-space data, we performed *k-t* BLAST and *k-t* PCA reconstructions using acceleration factors of 2, 4, 8, and 16 with 11 training profiles. For *k-t* PCA we used 15 principal components to calculate $\mathbf{B}$. In addition, we repeated the experiments where the frame order of the original data had been randomized. This creates more rapid signal intensity changes over time, thereby increasing the temporal resolution of the data.

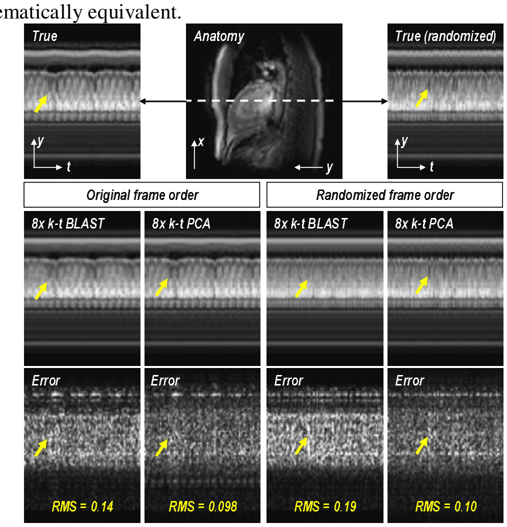

FIG. 1. Reconstruction of real-time cardiac data using 11 training profiles and 8x acceleration. Clearly, *k-t* PCA reconstructs the data with random frame order (i.e., higher temporal resolution) more accurately than *k-t* BLAST.

RESULTS: Figure 1 shows example reconstructions with 8x acceleration and 11 training profiles of a time profile aligned through the centre of the heart. For the original frame order, and for the randomized frame order in particular, *k-t* PCA reconstructs the data more accurately than *k-t* BLAST. This overall behaviour is also seen for other acceleration factors, as indicated by the mean reconstruction (RMS) errors in Fig. 2. Interestingly, the RMS error for *k-t* PCA is independent of the frame order, indicating that the proposed technique exhibits less temporal smoothing than *k-t* BLAST. However, for 2-fold data acceleration, *k-t* BLAST is consistently more accurate than *k-t* PCA. This is because the aliasing in *x-f* space is relatively easy to resolve for 2-fold acceleration [3], combined with the fact that *k-t* BLAST has more degrees of freedom than *k-t* PCA.

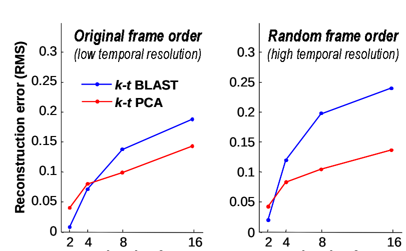

FIG. 2. Relationship between acceleration factor (with 11 training profiles) and the RMS error. Notice that the RMS error of *k-t* PCA is independent of the frame order.

CONCLUSIONS: We have presented the theory of *k-t* PCA and compared its performance with *k-t* BLAST based on simulations of real-time cardiac images. Both methods can be viewed conceptually in the framework of *k-t* PCA simply by a change of temporal basis. Specifically, *k-t* BLAST uses the unit basis (i.e., $\mathbf{B} = \mathbf{I}$), whereas *k-t* PCA utilizes a temporal basis tailored to the training data. The advantage of the unit basis is that it has more degrees of freedom which, in principle, allow modeling of arbitrary dynamic objects. Therefore, *k-t* BLAST is more accurate at 2-fold acceleration. However, since the signal encoding equation is always underdetermined, *k-t* BLAST reconstruction relies entirely on the estimated signal covariance, which in turn compromises temporal resolution. On the other hand, *k-t* PCA constrains the shape of the temporal frequency content, inducing an interdependence between the signal at different frequency components. This results in a signal encoding equation that is effectively overdetermined, which in turn improves the temporal fidelity compared to *k-t* BLAST. Similar to *k-t* SENSE, *k-t* PCA can also incorporate sensitivity encoding [4], which may add to the stability of the solution. In general, the presented framework allows incorporating any prior knowledge in the form of temporal basis functions. For real-time cardiac imaging, such temporal basis functions could be derived from respiratory navigators (or respiratory bellows) and the patients electrocardiogram [5].

REFERENCES: 1) Tsao et al. MRM 2003, 2) Tsao et al. MRM 2005, 3) Madore et al. MRM 1999, 4) Pruessmann et al. MRM 1999, 5) Glover et al. MRM 2000.