Reciprocity Relations in Travelling Wave MRI

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Introduction: Signal generation in NMR experiments, i.e., exciting and detecting transverse magnetization by electromagnetic interaction, can be described in the framework of reciprocity between the exciting RF fields and the received signal. Due to the coupling of the spins to the main magnetic field, the reciprocity of the NMR signal differs from the common Lorentz formulation for electromagnetic systems with linear isotropic materials, effectively by breaking the isotropy of the material. In NMR one is confronted with the more intriguing form of generalized reciprocity [1] or gyrotropism [2,3]. The latter have been well confirmed experimentally, especially by comparison with the predictions made by simulations of RF coils. However, some new flavour of reciprocity considerations has recently been prompted by the suggestion of travelling-wave coupling between the MRI probe and spins in the sample [4]. Studying the reciprocity of travelling-wave MRI is the aim of the present work.

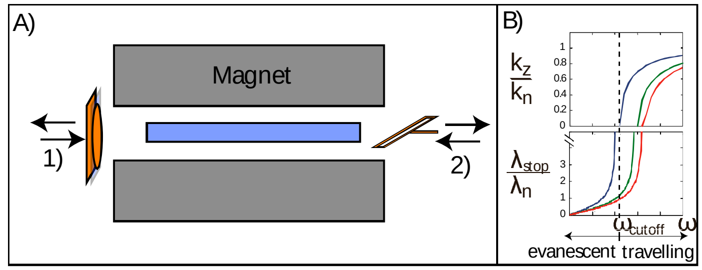

Figure 1: A) setup used, B) Behavior of axial wavelength of the modes inside the waveguide

Theory: It has been shown that propagating RF waves can be established in a human MRI system allowing remote detection of NMR signals [4]. In such a setup, a conductive lining inside the bore acts as a waveguide. The modes of such a waveguide are fed by an antenna positioned at the end of the bore, establishing a propagating mode inside the waveguide. In an axially uniform case, the field of each mode is characterized by its transverse field distribution (E_T, B_T) and by the longitudinal wave vector of the mode k_z determining the axial phase propagation. The transverse field distribution and the dispersion of k_z are characteristic for each mode (as shown in Fig.1B for the first three modes of a circular waveguide). Hence the fields and especially the for NMR relevant circularly polarized magnetic field components (B_1^+, B_1^-) exhibit a linear phase change along the axis of the bore (see Eq. 1&2). According to the reciprocity relation of NMR signals, the phase of the signal measured using the antenna at one end of the bore should hence show twice the phase change in the sample along the axis (Eq. 3&4). The sign of the phase gradient is dependent on the side of the bore the antenna is located relative to the main magnetic field direction. In the case, the irradiating and receiving antenna are located on opposite sides of the magnet, no linear phase change along the waveguide is expected in the NMR signal.

Methods & Results: In order to record the RF induced phase, the phase induced by local off-resonances was subtracted using two acquisitions with different echo times (TE=3.1 ms/3.6 ms). In a first experiment the dependence of k_z on the dielectric loading of the waveguide was measured. Two water filled cylinders of 4 cm and 7 cm diameter and a length of 1.5 m were centred in the bore and measured using a RF spoiled gradient echo sequence exciting and receiving using a linearly polarized patch antenna at the front end of the bore (antenna 1) in Fig.1A). Figure 2.A&B show the magnitude images over a FOV of 50 cm. Figure 2.C shows the phase recorded along the lines marked in Fig.2.A&B for the two cylinders. The negligible phase change along the small cylinder suggests that the NMR frequency is very close to the cut-off of the dielectrically filled waveguide. Hence, the excitation and reception is in-phase along the bore (k_z=0). In the case of the large cylinder, there is a clear linear trend in the phase. In order to study multi-port reciprocity relations of NMR signals, the experiment was performed for all combinations of sending and receiving with two antennas located at opposite ends of the waveguide. Figure 2.D) shows that the direction of the phase gradient changes with the direction of irradiation and detection if using the same antenna ($S_{1\rightarrow1}$, $S_{2\rightarrow2}$). This phase gradient vanishes when transmitting and receiving from different ends of the waveguide ($S_{1\rightarrow2}$, $S_{2\rightarrow1}$). The phase plots in Fig.2 were restricted to the linear range of the gradients avoiding any corrections on the data.

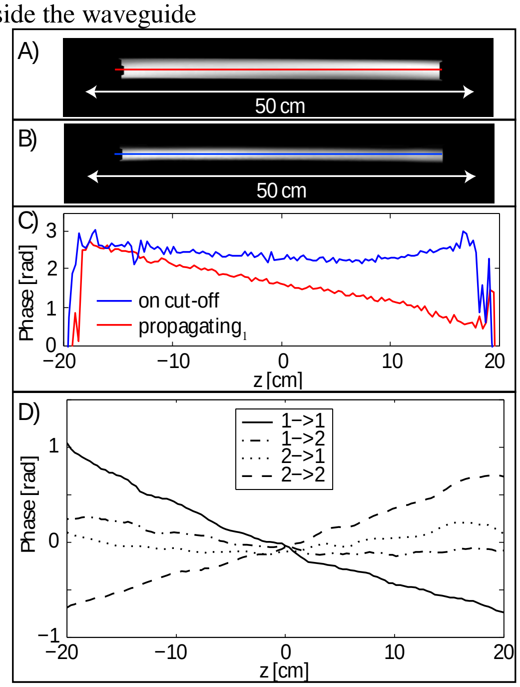

Figure 2: A) gradient echo magnitude image of thick and B) thin cylinder. C) Axial phase distribution in A) and B). D) Axial phase distribution recorded by transmitting and receiving with two antennas from opposite sides by turns.

$$B_{1,1}^{+}(z) = e^{-ik_z z}\tilde{B}_T^{+}(x,y); \quad B_{1,1}^{-}(z) = e^{ik_z z}\tilde{B}_T^{-}(x,y) \quad (1)$$

$$B_{1,2}^{+}(z) = e^{ik_z z}\tilde{B}_T^{+}(x,y); \quad B_{1,2}^{-}(z) = e^{-ik_z z}\tilde{B}_T^{-}(x,y) \quad (2)$$

$$\Rightarrow S_{1\rightarrow1}(z) \propto B_{1,1}^{+}(B_{1,1}^{-})^{*} = e^{-i2k_z z}\tilde{B}_T^{+}(x,y)\tilde{B}_T^{-*}(x,y) \quad (3)$$

$$S_{2\rightarrow2}(z) \propto e^{i2k_z z}\tilde{B}_T^{+}(x,y)\tilde{B}_T^{-*}(x,y) \quad (4)$$

$$S_{1\rightarrow2}(z) \propto B_{1,1}^{+}(B_{1,2}^{-})^{*} = \tilde{B}_T^{+}(x,y)\tilde{B}_T^{-*}(x,y) \quad (5)$$

$$S_{2\rightarrow1}(z) \propto \tilde{B}_T^{+}(x,y)\tilde{B}_T^{-*}(x,y); \quad (6)$$

Conclusion: It could be shown that the phase of the NMR signal using multiple antennas coincides with the expected axial phase gradient of the transverse magnetic field of the waveguide mode using MR reciprocity relations. This shows accordance to reciprocity relations found for NMR which was so far confirmed for experiments conducted in the near-field inductive regime. The phase relation found in between the signals recorded can be related to the RF phase delay differences the signal is picking up between the transmitting antenna and the spin and its way to the receiving antenna for different locations inside the waveguide. This delay could readily be used for signal encoding in traditional parallel imaging schemes by using receivers at both ends of the waveguide.

[1] Hoult, Conc Magn Res, 12 (2000) [2] Ibrahim, Magn Res Med 54 (2005) [3] Tropp, Phys Rev A, 74 (2006) [4] Brunner et al. ISMRM 434 (2008)