

# Information Theory in MRI

D. O. Brunner<sup>1</sup>, J. A. Nordmeyer-Massner<sup>1</sup>, and K. P. Pruessmann<sup>1</sup>

<sup>1</sup>Institute for Biomedical Engineering, University and ETH Zurich, Zurich, Zurich, Switzerland

**Introduction:** Benchmarking of MRI acquisitions methods and devices, such as k-space trajectories and coil arrays is usually based on SNR and speed considerations. E.g. for comparison of receive coil arrays the baseline SNR in an image is considered together with g-factors for evaluation of parallel imaging performance [1,2]. The flaw of these approaches is that the baseline SNR and the g-factor are trajectory and reconstruction dependent. This adds unwanted priors to the measurement whose influence on the results is in general hard to determine. The goal of this work is to find a general holistic metric for MRI data acquisition allowing a prior free benchmarking.

**Theory:** One of the main results of information theory is the ability to calculate the information rate that can be broadcasted via signal  $\mathbf{X}$  (see Fig. 1) to the received signal  $\mathbf{Y}$  using unit bandwidth [3]. This rate has been proven by Shannon to be limited by the mutual information channel capacity ( $I(\mathbf{X};\mathbf{Y})$ ). In the case of additive white Gaussian noise channels this is expressed by Eq.1. The matrix  $\mathbf{K}_Y$  represents the correlation between the signals of the different channels of the array and  $\Psi_N$  their noise correlation. The output of Eq.1 has the unit bits/s per unit bandwidth and gives the maximum information possibly received during any experiment. By itself, this represents an efficiency-measure of the acquisition including the coil and any encoding paradigm independently of any image reconstruction. Eqs.2 show the calculation of the information transport from the spins ( $\rho$ ) encoded by E during scanning (e.g. by means of gradients  $E=e^{ikr}$ ) to the receiver with its noise contribution  $N_r$ . The spins are detected by the sensitivity of coil  $i$  ( $B_{i,j}$ ) (stacked in the matrix  $\mathbf{H}$ ) concomitantly with the noise of the coil ( $N_a$ ), that is added before the signal is sent throughout the matching network  $\mathbf{P}$  to the before mentioned receiver. Using this formalism, the mutual information (Eq.1) of every module of the receive chain can be evaluated independently, revealing its performance.

**Examples:** 1. The Roemer SNR [1] (neglecting preamp noise and matching for simplicity) can be expressed in terms of the receive matrix  $\mathbf{H}$  and the noise covariance assuming the input image is fully resolved ( $\mathbf{K}_X=\mathbf{1}$ ). In order to compare the two expressions we transform them into their eigensystem (where  $\mathbf{\Lambda}$  and  $\mathbf{Q}$  represent the Eigen decomposition of  $\mathbf{H}^H\mathbf{H}\Psi_N^{-1}=\mathbf{Q}^H\mathbf{\Lambda}\mathbf{Q}$  in Eq.4). Due to the log function the mutual information expression values the orthogonality of the signal among the channels in addition to the mere SNR and is thus able to reflect the encoding capability of the array.

2. The benefit of a coil decoupling network ( $\mathbf{P}$ ) can readily be seen from Eq.3. Since  $\mathbf{P}$  acts on the noise and the signal correlation of the array equally, the information drawn from the array would be equal for every non-singular  $\mathbf{P}$ , as long as the preamplifiers are noise free. This means that the needed amount of decoupling between the coils is determined by the noise figure of the preamplifiers which is enhanced by the conditioning of  $\mathbf{P}$ .

3. The impedance transformation of an 8 channel wrist array coil [4] was varied experimentally from providing maximum preamplifier decoupling (see Fig. 2, setting 11). Although maximum SNR was not achieved with maximum decoupling but with setting 21, the maximum mutual information was received by 11 (see arrow 1). This discrepancy is explained by the lower g-factor of 11 reflecting the spatial encoding capability of the array. A similar behavior is marked also by arrow 2.

**Conclusion:** Information theory provides metrics for benchmarking MRI signal acquisition in a holistic manner valuating SNR and encoding performance equitably. It allows to measure the performance of an array alone or in conjunction with different encoding schemes without any priors on reconstruction and gives an upper boundary on the efficiency of the acquisition and reconstruction process.

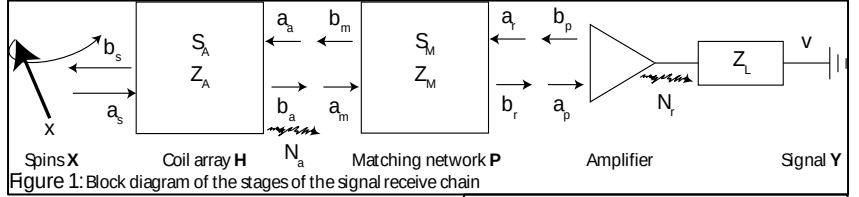


Figure 1: Block diagram of the stages of the signal receive chain

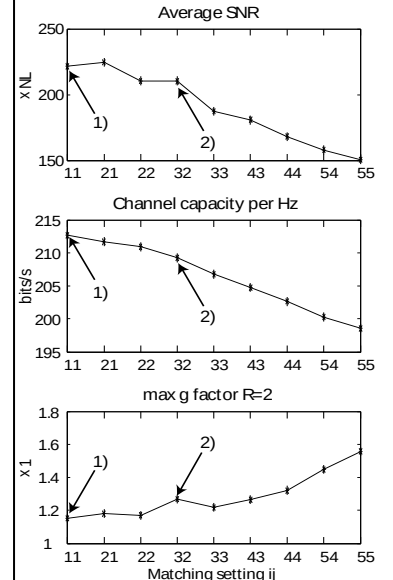


Figure 2: Results from matching optimization of an 8 channel array revealing peak performing setting unambiguously

$$I(\mathbf{Y};\mathbf{X}) = \log_2 \left( \frac{\det(\mathbf{K}_Y + \Psi_N)}{\det(\Psi_N)} \right) \quad (1)$$

$$v_i(t) = \sum_j \mathbf{P}_{i,j} \left( \int B_{i,j}^-(\vec{r}) \rho(\vec{r}, t) E(\vec{r}, t) d\vec{r} + N_{a,j}(t) \right) + N_r(t); \text{ Recorded signal}$$

$$\mathbf{K}_X(\vec{r}, \vec{r}') = \frac{1}{T} \int_0^T \rho^*(\vec{r}', t) E^*(\vec{r}', t) \rho(\vec{r}, t) E(\vec{r}, t) dt; \text{ Spin signal}$$

$$(\mathbf{K}_A)_{i,j} = \iint B_{i,i}^*(\vec{r}') \underbrace{B_{i,j}^-(\vec{r})}_{(\mathbf{H})_{p,j} := B_{i,j}^-(\vec{r}_p)} \mathbf{K}_X(\vec{r}, \vec{r}') d\vec{r} d\vec{r}' = \mathbf{H}^H \mathbf{K}_X \mathbf{H}; \text{ Coil signal cor.} \quad (2)$$

$$(\Psi_{N_a})_{i,j} = \frac{1}{T} \int_0^T N_{a,i}^*(t) N_{a,j}(t) dt = 4k_B TR(\mathbf{1} - \mathbf{S}_A^* \mathbf{S}_A); \text{ Array noise correlation}$$

$$(\mathbf{K}_Y)_{i,j} = \mathbf{P}^H \mathbf{K}_A \mathbf{P}; \text{ Receive signal}; (\Psi_{N_r})_{i,j} = \mathbf{P}^H \Psi_{N_a} \mathbf{P} + \Psi_{N_r}; \text{ Received noise}$$

$$I(\mathbf{Y};\mathbf{X}) = \log_2 \left( \frac{\det(\mathbf{P}^H \mathbf{K}_X \mathbf{P} + \mathbf{P}^H \Psi_{N_a} \mathbf{P} + \Psi_{N_r})}{\det(\mathbf{P}^H \Psi_{N_a} \mathbf{P} + \Psi_{N_r})} \right) = \log_2 \left( \frac{\det(\mathbf{K}_X + \Psi_{N_a} + (\mathbf{P})^{-1} \Psi_{N_r} (\mathbf{P}^H)^{-1})}{\det(\Psi_{N_a} + (\mathbf{P})^{-1} \Psi_{N_r} (\mathbf{P}^H)^{-1})} \right) \quad (3)$$

$$\text{SNR}_{\text{Roemer}}^2 = \text{Tr}(\mathbf{H}^H \mathbf{H} \Psi^{-1}) = \sum_j \Lambda_{jj} \quad (4)$$

$$I(\mathbf{K}_X = \mathbf{1}) = \log_2(\det(\mathbf{H}^H \mathbf{H} \Psi^{-1} + \mathbf{1})) = \sum_i \log_2(\Lambda_{ii} + 1)$$

[1] Roemer et al. *MRM* 16, (1990). [2] Pruessmann et al. *MRM* 42, (1999) [3] Ezio Biglieri, Coding for Wireless Channels, Springer 2005 [4] Nordmeyer-Massner et. al., *MRM*2008 (in press),