# Field Superposition Method for RF coil design

## J. Paska<sup>1</sup>, J. Froehlich<sup>1</sup>, D. O. Brunner<sup>2</sup>, K. P. Pruessmann<sup>2</sup>, and R. Vahldieck<sup>1</sup>

<sup>1</sup>Laboratory for Electromagnetic Fields and Microwave Electronics, ETH Zurich, Zurich, Switzerland, <sup>2</sup>Institute for Biomedical Engineering, ETH Zurich, Zurich, Switzerland

### Introduction

RF coils for MRI are designed to achieve a desired field distribution (e.g. high and homogeneous  $B_1^+$ , low E-fields, low noise and a high  $B_1$ ). At ultra-high fields (>4 T), the radio frequency (RF) E and H-fields are coupled and the interactions with the imaging object becomes more complex [1], [2]. The prediction of the electromagnetic field distribution induced by a specific RF-coil configuration by numerical means therefore requires the full wave solution of Maxwell's equations. For a given scenario including a coil geometry with lumped elements at fixed positions and a load, many iterations are necessary to find the optimal values for the lumped elements (e.g. capacitors) and the positioning of the feeding port in order to achieve the desired EM field. For complex loads, such as human models, one single simulation can take several hours of computation time.

Therefore, a new method allowing for faster calculation of the EM fields associated with a specific RF coil configuration including variable lumped elements is needed. The proposed field superposition method (FSM) reduces the number of required full wave EM simulations. The fields for a specific set of values for the lumped elements can be calculated in a post-processing step very quickly. For N variable lumped elements on a coil configuration, only N full wave simulations have to be computed.

### Theory

In FDTD, FIT and FEM based full wave EM simulations methods the computational domain is discretized. Lumped elements are represented by edges (or faces) of cells. Instead of giving these edges a fixed value, they can be left variable. Each of these edges can be excited with a source and their transfer function towards each other and to every point in space can be measured. Knowing the transfer function of every variable edge a linear matrix equation can be found that allows to replace the variable edges by arbitrary lumped elements and to compute the resulting EM field in a post-processing step very quickly. If the variable edges are excited with voltage sources having a source impedance of 50 Ω, the E-field of a single RF-coil at an arbitrary position x in space can be calculated according to:

$$E = ([U] - [S])^{-1} \cdot [E]^{50} \cdot ([U] - [S]) \cdot ([U] - [\tilde{S}][S])^{-1} \cdot [b]_{g} \quad (1)$$

where [U] denotes the NxN unitary matrix,  $[\tilde{S}]$  the NxN scattering matrix having the reflection coefficients of the fixed lumped elements in the variable edges as diagonal elements.  $[E]^{50}$  is a 1xN vector, having the E-field that the *i*th variable edge excited by a 50  $\Omega$  source with a forward power of 1W (a=1), while all other ports are terminated with a 50  $\Omega$  resistance.  $[b]_g$  is a Nx1 source vector, filled with zeros, except for one entry corresponding to the excitation port, the corresponding element in the  $[\bar{S}]$  matrix is the source impedance. The NxN matrix [S] is the scattering matrix between the variable edges.

This method can be easily extended to calculate the EM-fields of coil arrays, consisting of M coils, by extending the source vector to a NxM source matrix, with the *j*th source vector in the *j*th column. This results in a 1xM vector [E] on the left side of eq. (1), the *j*th component being the field  $E_j$  produced by coil j. Thus only N simulations have to be performed to determine the fields [E]<sup>50</sup> and the scattering matrix [S].



Figure 1: Simulation setup, microstrip under phantom



Figure 3: H-field, sweep of capacitor C5

#### <u>Methods</u>

To validate the FSM a phantom simulating the human head and shoulder region excited by a single microstrip RF coil including five 50  $\Omega$  sources was simulated. The coil is placed next to the head above the shoulder. The numbering of the ports and the simulation setup is shown in Fig. 1.

The setup was simulated using a FITD based simulation program (CST Microwave Studio). The phantom was filled with a tissue simulating liquid ( $\sigma$ =0.91 S/m, and  $\epsilon_r$ =61).

The EM fields were computed at the Larmor frequency at 7T of two simulation setups. The first simulation used for the validation includes one 50  $\Omega$ -source at port one together with four fixed capacitances (C<sub>2</sub>=3pF, C<sub>3</sub>=5pF, C<sub>4</sub>=10pF, C<sub>5</sub>=1pF) placed at the corresponding ports. A second 5-port simulation with the same setup was performed using the novel FSM replacing the before mentioned four capacitors by 50  $\Omega$ -sources. In this case five simulations have to be carried out, with one port being excited at a time, while the other ports were terminated with 50  $\Omega$  resistances.

#### **Results**

The 1-port simulation lasted ~2 hours. The full wave EM simulation part of the 5-port simulation lasted about 10 hours for all 5 ports. The FSM post processing however lasted only fractions of a second.

The EM fields in Fig. 2 and 3 were plotted along the z-axis above the RF coil in the middle of the phantom. In all the plots the EM fields were normalized to an input power of 1 Watt.

Fig. 2 shows the magnitudes of the E- (in red) and the H-fields (in black) of the 1-port simulation (solid line) and the 5-port simulation using the same setup as the 1-port simulation applying the FSM (crosses).

Fig. 3 shows the magnitude of the H-field computed with the FSM using the 5-port simulation. The same setup as for the 1-port simulation was chosen, except for the capacitor  $C_5$  sweeping the capacitance value from 1 to11 pF in steps of 2 pF.

# **Discussion & Conclusion**

As shown in Fig. 2 the FSM method yields the same EM fields as conventional 1-port simulations. Once the 5port simulation is carried out an arbitrary set of lumped elements can be connected to the coil and the EM field can be evaluated very quickly. Only the S-matrix of the 5-ports and the EM fields that each source excites has to be known. The six graphs in Fig. 3 were obtained in less than a second. For comparison, simulating these results in full wave simulations would take 6\*2=12 hours. Fig. 3 shows an improvement in homogeneity as we increase the capacitance value C<sub>5</sub>. Different source positions could also easily be investigated.

Using the FSM, the actual EM field distribution of a given coil geometry including lumped elements at fixed positions can be optimized very quickly. This speeds up the RF coil design process significantly allowing extensive optimization procedures being performed in a short time. These optimizations can also be applied for coil array detuning or coil array decoupling as well as for the analysis of potential variations in field strengths for safety considerations (malfunction of components etc.).

## **References**

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