Novel Algorithm for l1 Wavelet-Based MR Image Reconstruction

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Introduction

In fast MR imaging, reconstruction artifacts due to undersampled k-space can be greatly reduced by applying proper nonlinear reconstructions [1] based on image-sparsifying transforms. While state-of-the-art methods rely on total variation (TV), in this paper we propose to use wavelets instead, along with a very fast algorithm. Simulations and experimental results show our ability to reduce computational costs while maintaining SNR and image quality. We propose an iterative algorithm that also makes the technique computationally competitive. Our algorithm is versatile and can be used for any linear MR imaging problem, for instance SENSE [2].

Theory

Our algorithm is based on the recent Iterative Shrinkage/Thresholding Algorithm (ISTA) [3] and Fast ISTA (FISTA) [4] that consists of repeating the sequence: gradient descent and wavelet domain thresholding. FISTA represents the current state-of-the-art for solving the variational problem

$$\tilde{x} = \arg\min_x \ \|m - Ex\|_2^2 + \lambda \|Wx\|_1^1$$

where x is the image, m the k-space data, E the encoding matrix, and W the sparsifying transform (i.e., wavelets). Our contribution is to propose an acceleration scheme that is optimized for the MR reconstruction problem. More precisely, instead of a single parameter that is normally used to set the gradient steps and wavelet thresholds, we introduce several precomputed parameters that depend on the wavelet subband. They are mathematically determined to maximize convergence speed and depend on the k-space sampling as well as the wavelet transform [5].

Methods

Our implementation is done using Matlab™ 7.9 on a 64-bit 8-core computer, 4GB RAM, Mac OS X 10.6. For Fourier computations we use the NUFFT algorithm [6]. We compared the reconstruction using four methods: Tikhonov regularization using a conjugate-gradient (CG) algorithm, Total Variation with Iteratively Reweighted Least-Squares, and the proposed ℓ_1 regularization with Haar wavelets using ISTA and our method. For each method, we optimized the free parameters to minimize the reconstruction error. We introduced random shifting in our algorithm in order to reduce blocking artifacts.

The purpose of the first synthetic experiment is to estimate the computational costs. We simulated MR data for the 2D Shepp-Logan brain phantom and a spiral trajectory of 7 interleaves supporting a 168x168 reconstruction matrix. We added Gaussian noise with signal to noise ratio (SNR) 20dB. The reference image was a direct discretization of the phantom. The spiral trajectory, together with the Haar wavelet transform, favors our algorithm and leads to a sizeable acceleration over FISTA. This piecewise-constant phantom benefits mostly to TV regularization.

In the second experiment, we used textured data to test the possibility to reconstruct undersampled data obtained with a single receiver coil. Cartesian gradient-echo with a matrix of 200x200 imaging was conducted on kiwi fruit in a 3T Achieva system (Philips Medical Systems, Best, The Netherlands). A field camera was used to measure the exact k-space trajectories of this scan [7]. Five sets of data corresponding to Cartesian trajectories designed for a 200x200 matrix were acquired, with increasing reduction factors 1, 10/9, 5/4, 10/7 and 5/3 in the phase-encoding direction. The reference image was obtained using conventional reconstruction from the full data, which contains 10x more samples than required by Nyquist. For reconstruction, we used the trajectory with undersampling R=5/4 and added full sampling of the 8% central k-space to stabilize reconstructions.

Results

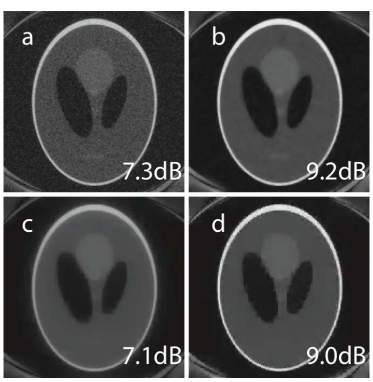

Fig. 1 *Best SNR reconstructions after 2s with CG (a), TV (b), ISTA (c), and our method (d).*

We give in Fig. 1 the reconstructions obtained for the first experiment after 2s. The four algorithms described in Section “Methods” were employed. In Fig. 2 we observe the time evolution of the reconstruction SNR compared to the reference image. Note that TV performs a bit better than our method in terms of SNR, mainly due to the absence of texture in the Shepp-Logan phantom. Compared to ISTA, our algorithm is 6x faster in reaching the performance of Tikhonov regularization.

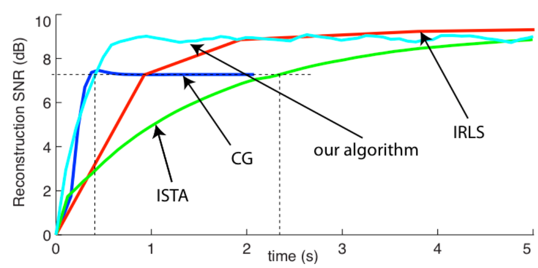

Fig. 2 *Time evolution of the reconstruction SNR.*

We illustrate in Fig. 3 reconstructions obtained for the second experiment. We observe that the reconstructions are not completely artifact-free. However, we measured a better reconstruction SNR with our method (15.7dB) than with TV regularization (14.7dB) or Tikhonov regularization (12.5dB). In terms of computational speed, our method was equivalent to FISTA. Compared to ISTA it offers a 3.7x speedup (i.e. 4.5s vs. 17s) to reach 12.5dB and 6x speedup to reach 14.7dB (7s vs. 42s).

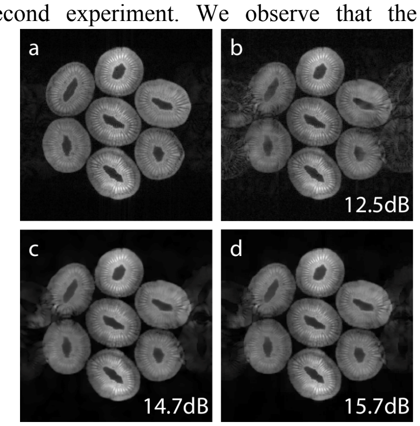

Fig. 3 *Best SNR reconstructions with CG (a), TV (b), ISTA (c), and FISTA (d).*

Conclusions

The proposed wavelet-based algorithm yields reconstructions that are competitive with TV regularization in terms of quality, in particular when data are textured. Further validations will be presented in the future to strengthen this claim. The algorithm parameters are optimized for the reconstruction problem, accelerating reconstructions up to 6 times compared to alternatives.

References:

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