Rapid relative pressure map computation from velocity-encoded phase-contrast measurements

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Introduction

Spatially resolved assessment of blood pressure dynamics is desirable for the evaluation of various cardiovascular disorders. For example, the local change of pressure across vascular or valvular stenoses can be used to characterize the severity of the constriction. For the heart chambers, systolic and diastolic efficiency is directly related to the dynamics of the respective filling pressures. Time-resolved velocity data acquired by velocity encoded phase-contrast MRI can be used to derive relative pressure [1,2]. Based on the Navier-Stokes equation relating a 3D velocity vector field to the pressure gradient vector field for an incompressible fluid, taking the pressure divergence and neglecting body forces, a pressure Poisson equation can be formulated [3]. Solving this Poisson equation allows to calculate the relative pressure field, avoiding the noise amplification and path dependence of directly integrating pressure gradients fields. However, the implementation of a solver for the pressure Poisson equation for arbitrarily shaped domains is not straight-forward and boundary conditions have to be chosen carefully. Recently, a multi-grid solver has been proposed for improved and accelerated relative pressure field computation [4].
In this paper, we present an alternate solver for the pressure Poisson equation using a direct generalized minimal residual procedure (GMRES) [5].

Methods

Pressure Poisson equation solver: Assuming laminar flow conditions and an incompressible Newtonian fluid with density ρ and viscosity μ the simplified Poisson equation relates the pressure p to the velocity field V according to:

$$\nabla^2 p = \nabla\left(-\rho\frac{\partial V}{\partial t} - \rho V\cdot\nabla V + \mu\nabla^2 V\right) \qquad [1]$$

Equation 1 is discretized using central differences for the divergence calculations. The Laplacian is calculated using a component-wise symmetric 3-point discretization on inner voxels and a linear extrapolation on boundary voxels. The resulting system of linear equations is solved using an iterative Krylov subspace approach where in each iteration the residual norm is minimized. This so-called GMRES procedure includes an Arnoldi iteration and for each iteration step an inner least squares problem is solved. A maximum of 50 inner and 10 outer iteration steps were performed until the residual converged below a predefined tolerance. Boundaries were assumed to be static and impermeable by enforcing that the flow into the boundary always remained zero. Consistently, Neumann boundary conditions were chosen for solving Eq. 1 [6]. The boundary conditions were directly incorporated into the differential operators such that they remain implicitly fulfilled when solving Eq. 1. To keep the Laplacian operator on the left hand side of Eq 1 linear, the corresponding fixed additive boundary terms entering the operator were subtracted from the right hand side of Eq. 1 for boundary points. The solver was implemented entirely in C++ using double precision floating-point arithmetic.
4D flow imaging: Velocity vector field measurements in the aortic arch of healthy volunteers (N=3) and patients (N=2) were performed using time-resolved 3D phase-contrast imaging with a prospectively ECG-triggered, T1-weighted, segmented gradient echo sequence. Nominal spatial resolution was 1.4x1.4x3mm^3, TR and TE were 4.5 ms and 2.7 ms, and the flip angle was 6°. Velocity encoding in all directions was 150 cm/s (300 cm/s in patients). Twenty-five cardiac phases were acquired uniformly distributed over the cardiac cycle. Images were reconstructed to a 128x128x17 image matrix. Navigator-based respiratory gating was used and the total acquisition time was approximately 16 minutes depending on gating efficiency and heart rate. Correction of the data for the effects of concomitant gradient fields was performed on the scanner. A linear phase correction for eddy current effects was achieved by subtracting a least-squares fit to the phase distribution in stationary tissue.
Volume segmentation: The volume of interest used as the computational domain was segmented by thresholding a velocity amplitude dataset in late systole weighted with the signal magnitude. Relative pressure fields in all cardiac phases were computed from the same volume, neglecting motion of the aorta over the cardiac cycle.

Results

For all cardiac phases, the solver converged to a solution for all datasets. Calculation times were typically below 4s per cardiac phase on a 2.5 GHz Core Duo Intel CPU. The resulting maps showed a smooth pressure distribution across the aortic arch. In the healthy volunteers pressure distributions in the ascending aorta were larger compared to those seen in the descending aorta in early systolic phases and vice-versa in late systolic phases.

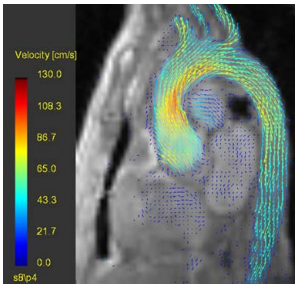

Fig 1a: Systolic flow velocity field in the aortic arch of a healthy volunteer.

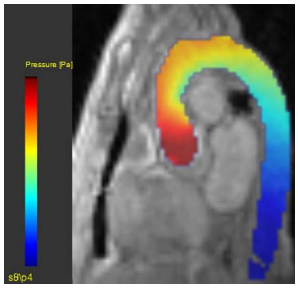

Fig 1b: Corresponding relative pressure map (color-coded in arbitrary units).

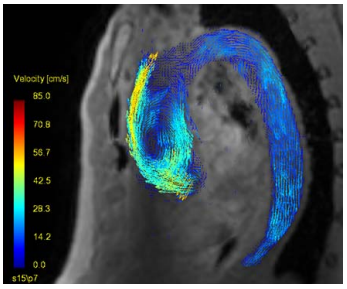

Fig 2a: Systolic flow velocity field in a patient with an aortic stenosis and dilated ascending aorta.

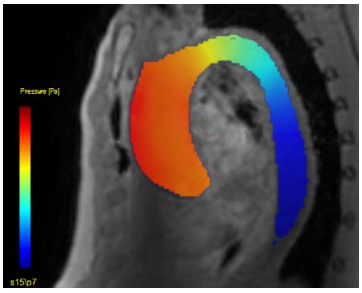

Fig 2b: Corresponding relative pressure map (color-coded in arbitrary units).

Discussion:

The proposed solver provides estimates of the relative pressure fields directly from segmented velocity field data acquired by velocity-encoded phase-contrast MRI. However, the automatic segmentation of the aorta proved difficult especially in the patient data. Segmentation errors significantly influenced the resulting relative pressure maps. Another potential problem is the excessive memory usage of the GMRES algorithm in very large computational domains. This can be addressed by an extension of the GMRES algorithm, whereby the method is restarted after a limited number of iterations. A restarted GMRES algorithm may be required for datasets with higher spatial resolution and matrix sizes. Also, appropriate preconditioning of the system may further optimize algorithm convergence and remains to be investigated.

References

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