MR Image Reconstruction Exploiting Nonlinear Transforms

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Introduction: In compressed sensing (CS), incoherent undersampling artifacts are removed by nonlinear denoising in sparse transform domains while ensuring consistency with the acquired k-space data. CS image reconstruction algorithms typically employ wavelet and finite differences transforms which both fulfill the restricted isometry property (RIP) [1], i.e. they are (nearly) orthogonal transforms. However, both transforms require a considerable amount of linear coefficients to model long-range correlations in images (Fig. 1), i.e. the sparsity is reduced. Recently, generalizations of RIP and CS reconstruction from nonlinear observations have been described [2].

In the present work, MR image reconstruction exploiting nonlinear transform domains is proposed. Nonlinear basis functions are employed for efficient representation of long-range correlations. An implicit nonlinear mapping of image blocks into a high-dimensional kernel feature space is derived from the neighborhood of the blocks using kernel principal component analysis (kernel PCA) [3]. Nonlinear denoising is achieved by projection onto the first principal components. Image reconstruction is performed using an iterative thresholding scheme [4] interleaving nonlinear projection and gradient updates for consistency with the acquired k-space data.

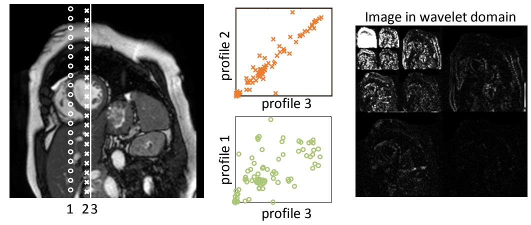

Figure 1: Middle graphs: Plots of the profiles indicated by circles and crosses against the solid line. Right image: 3-level wavelet transform of the left image. The graphs and the image in wavelet domain demonstrate the decreased sparsity when representing long-range correlations.

Theory: Kernel PCA comprises of three steps including a nonlinear mapping into a feature space (1), a linear PCA for separation of image data from artifacts (2), and a numerical back-mapping to input space (3). The nonlinear mapping is performed implicitly by means of Mercer's theorem, which states that positive-definite, symmetric kernel functions $k: \mathrm{X} \times \mathrm{X} \rightarrow \mathbb{R}$ can be written as scalar product: $k(\mathbf{x},\mathbf{y}) = \Phi(\mathbf{x})^T \Phi(\mathbf{y})$. Using a Gaussian kernel function $k(\mathbf{x},\mathbf{y}) = \exp(-0.5\|\mathbf{x}-\mathbf{y}\|_2^2 / \sigma^2)$, the feature space has infinitely many dimensions with data-dependent nonlinear basis functions. The linear PCA of step (2) is performed in a dot product space yielding the N most significant principal components where N is the number of training samples $\{\mathbf{x},\mathbf{y},\ldots\}$ where each sample vector represents a stacked image block (Fig. 2). Back-mapping is performed with a fixed-point iteration scheme [5]. The kernel width σ controls the degree of nonlinearity [6]. For very small σ, a single basis function represents all image data including image artifacts. Thereby, sparsity in kernel feature spaces is only meaningful if the true image signal is separable from undersampling artifacts. MR image reconstruction was performed using projected Landweber updates. With encoding matrix **E**, k-space data $\boldsymbol{d}$, image estimate $\boldsymbol{x}_n$ and kernel PCA projection P, the iteration steps are given by $\boldsymbol{x}_{n+1} = \mathrm{P}(\boldsymbol{x}_n - \mathbf{E}^H(\mathbf{E}\boldsymbol{x}_n - \boldsymbol{d}))$.

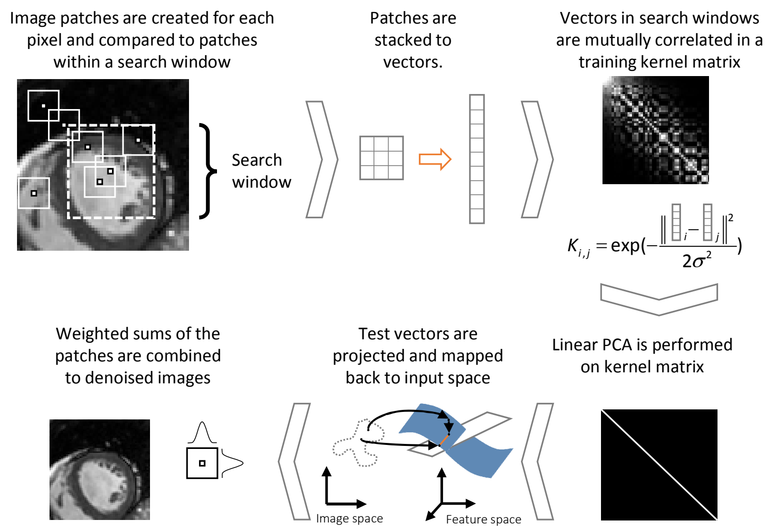

Figure 2: Flow chart for projections in a nonlinear kernel feature space using kernel PCA.

Methods: Fully sampled 2D and 3D cardiac images were acquired on a 1.5 T and 3T system (Philips Healthcare, The Netherlands) in subjects after written consent was obtained according to institutional guidelines. The data were undersampled in phase-encode directions using reduction factors of 4 (2D) and 8 (3D) using a variable density sampling [7]. Scan parameters for the 2D scan included a field-of-view of 270x270 mm^2 and isotropic in-plane voxel size of 1.4 mm. Coil array compression from 28 coils into 4 virtual coils was used [8]. The 3D whole-heart scan was acquired with a 5-channel cardiac coil with a field-of-view of 256x256x144 mm^3 and isotropic voxel size of 1.33 mm.

Results: Image reconstruction results comparing CS *L1* minimization in wavelet and finite-differences transform domains and the proposed projections in a nonlinear kernel feature space are presented in Figure 3. The 3D whole-heart scan was reformatted to show the right coronary artery. The sampling patterns are shown to the right of the in vivo images.

Discussion: MR image reconstruction exploiting nonlinear transforms has successfully been implemented using projections onto principal components in a high-dimensional kernel feature space, employing kernel PCA. Image quality was found to improve considerably relative to standard CS reconstruction with wavelet and finite-differences transforms. Further work is warranted to explore the limits of nonlinear transform domain reconstruction.

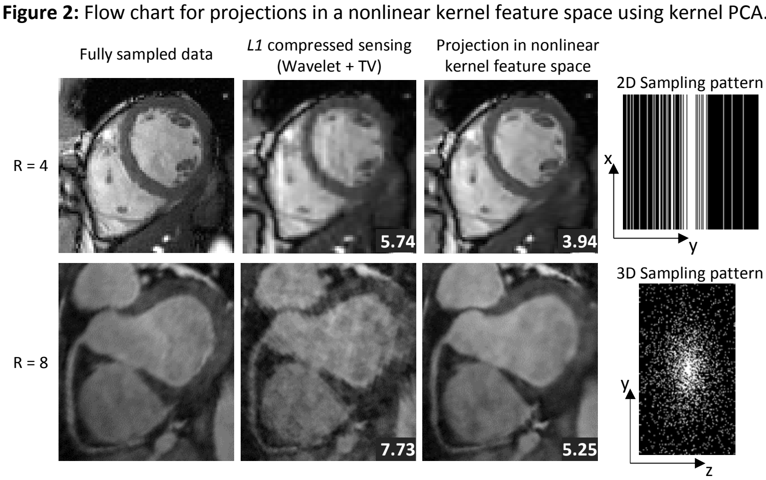

Figure 3: Image reconstruction results for 2D (upper row) and 3D (lower row) cardiac images comparing *L1* minimization in wavelet and finite differences transform domains with the proposed projections in a nonlinear kernel feature space. Root mean squared errors are indicated in the insets.

References: [1] Candes EJ, IEEE Trans. Inf. Theory (52), 2005, [2] Blumensath T, IEEE Trans. Inf. Theory (59), 2013, [3] Schölkopf B, Neural Comput (10), 1998, [4] Daubechies I, Comm. Pure Appl. Math. (LVII), 2004, [5] Mika A, Adv. Neural Inf. Process. Syst. (11), 1998, [6] Rasmussen PM, Neuroimage (60), 2012, [7] Lustig M, MRM (58), 2007, [8] Buehrer M, MRM (57), 2008