On the Quantification of Turbulent Kinetic Energy using Phase-Contrast MRI

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Purpose: Phase-Contrast (PC) MRI offers not only the possibility to quantify velocities, but also to assess the energy stored in turbulent flow[1]. It has been proposed to measure Turbulent Kinetic Energy (TKE) in the aorta as an indicator of severity of aortic stenosis[2]. The TKE signal model makes assumptions about a Gaussian distribution of velocities in a voxel, and the integral time scales which are associated with large-scale turbulence. Initial comparisons of the method with Computational Fluid Dynamics[3] and Particle Tracking Velocity[4] (PTV) showed good agreement. However, the validity of the assumption of Gaussian distribution, relevant turbulent time scales and voxel-size dependency have not been assessed so far. The objective of the present work was to validate the assumptions underpinning PC-MRI based TKE measurements using high-resolution PTV in a realistic aortic arch phantom.

Methods: Velocities in an elastic cast of an aortic arch (**Fig. 1a**) were measured using PC-MRI and PTV under steady flow condition (242 ml/s). PCI-MRI data were acquired on a 3T scanner (Philips Achieva, Philips Healthcare, Best, The Netherlands) using a 6-channel cardiac coil array and an isotropic resolution of 1.5 mm. A Bayesian Multipoint techique[5] was employed to achieve a high dynamic range for both velocity and TKE assessment. For the PTV measurements Rhodamine particles with a diameter of 200 µm were used, and 30 measurements with a high-speed camera (Fastcam SA5, Photron Ltd., Japan, 7000 frames/second) were recorded to achieve an isotropic resolution of 0.625 mm. TKE production P was calculated from PTV data according to:

$$P = -\overline{u'_i u'_j}\frac{\partial U_i}{\partial x_j} \qquad \tau_{TKE} = \frac{\int_V TKE}{P}$$

with U being the mean velocity, u' the velocity fluctuations and x the spatial position. The TKE turnover time τ_{TKE} was calculated by dividing the total TKE in the volume V by the production rate. The dependency of the PC-MRI signal magnitude |S| on the first moment of the velocity encoding gradient k_v is modeled according to:

$$|S(k_v)| = S_0 e^{-\frac{\sigma^2 k_v^2}{2}} \qquad TKE = \frac{\rho}{2}\sum_{i=1}^{3}\sigma_i^2$$

with σ^2 being the variance of the velocities within a voxel. Having acquired the variances of all 3 directions i, TKE is calculated by summation of σ^2 scaled with fluid density ρ.

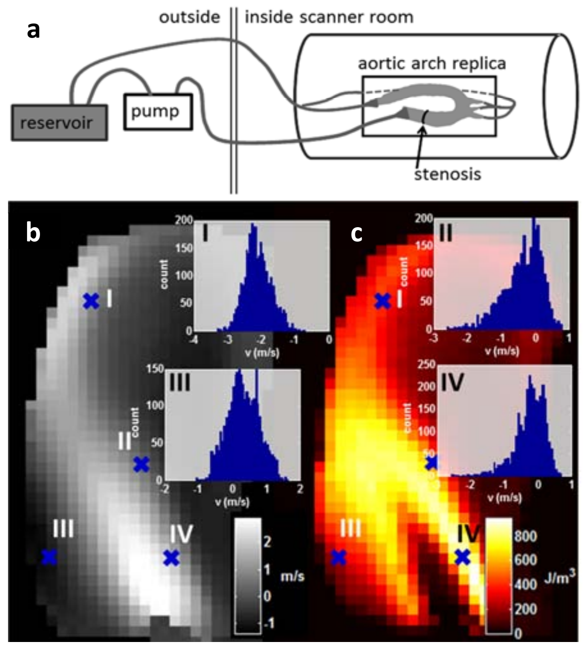

***Fig. 1**: Schematics of the flow phantom (a). Velocity (b) and TKE (c) maps as determined by PTV. Four example voxels (I-IV) are chosen to illustrate different velocity distributions at a voxelsize of 1.5 mm isotropic.*

Results: **Fig. 1b,c** shows the velocity component in feet-head direction, the TKE map and four example velocity distributions in different voxels. Maximum TKE at a resolution of 0.625 mm isotropic was 765 J/m³, highest velocity magnitude was 3.9 m/s. TKE turnover time τ_{TKE} was 71 ms. The integral time scale of large-scale and small-scale (Kolmogorov) turbulence[6] were found to be 50 and 2.3 ms, respectively. **Fig. 2a,b** illustrate the voxelwise correlation between MRI and PTV measurements, total relative error was -16.8 %. **Fig. 2c** shows the errors caused by increasing voxel size and deviation from the assumed Gaussian velocity distribution.

Discussion: The PTV data show that velocity distributions in different flow regimes do not strictly follow a Gaussian distribution. However the resulting underestimation of TKE of about 6% is still acceptable (**Fig. 2c**). In contrast, increasing voxel sizes lead to an *overestimation* of TKE levels caused by a broadening of the velocity distributions due to spatial velocity gradients. The total error level of PC-MRI measurements indicate a minimum resolution of 2 mm for error levels below 5%. However, the comparison of the actual MRI measurement (**Fig. 2a,b**) exhibit a higher error level and a systematic overestimation of low TKE values, and an underestimation of higher TKE values. This finding may be attributed in parts to the fact that the integral time scales of turbulence range from 2.3 to 50 ms, which might violate the assumption of τ >> TE. Misregistration could also lead to systematic errors, however they would likely affect the velocity correlation as well. The TKE turnover time of 71 ms suggests that for accurate estimation of energy loss during the cardiac cycle, peak TKE values alone are not sufficient and the variations in systolic duration between subjects have to be considered.

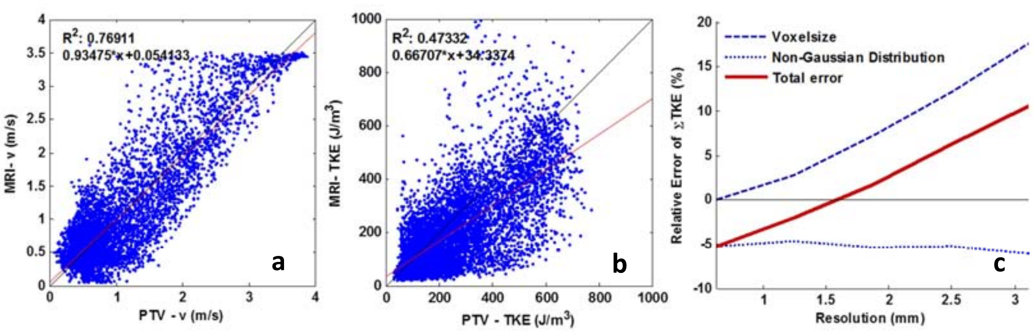

***Fig. 2**: Voxel-wise correlation between MRI and PTV of velocity magnitude (a) and TKE (b). Simulation results of the expected error resulting from Non-Gaussian velocity distributions within a voxel, and error dependence on resolution are presented in (c). Total error for both effects combined is also stated.*

References: [1]Dyverfeldt et al., J Magn Res Imag 2008, 28:655-663.
[2]Dyverfeldt et al., JACC Cardiovasc Imaging. 2013 Jan;6(1):64-71.
[3]Dyverfeldt et al., Magn Reson Imag 2009, 27:913-922.
[4]Knobloch et al., Magn Reson Med. 2013 May 13.
[5]Binter et al., Magn Reson Med. 2013 May;69(5):1337-45.
[6]Pope SB, Turbulent Flows, Cambridge University Press, 2000.