

# Image reconstruction from randomly undersampled cine MRI data - a comparison with the $k$ - $t$ BLAST method

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## Introduction:

Dynamic images of typical objects in MRI are highly redundant in space and time. Methods have been proposed which exploit this spatiotemporal correlation to allow speeding up the data acquisition process by undersampling on a regular grid in  $k$ - $t$  space (1). With  $k$ - $t$  BLAST temporal blurring might be introduced due to partial volume effects in the training data. At low acceleration factors the blurring of coarse structures is subtle but it can become noticeable for instance in valve imaging where small structures move at high temporal frequencies. To reduce the partial volume effect in the training data, the number of training profiles can be increased at the cost of a lower net acceleration factor.

Recently, a new method was proposed that allows an *exact* image reconstruction from randomly undersampled Fourier samples (2) by minimizing the  $L^1$ -norm of the image in a sparse domain. The main result of (2) states, that for a signal  $f(t)$ ,  $t \in T$  which is sampled on a *nonuniform, random* sampling domain  $\Omega \subset T$  the missing coefficients  $f(t)$ ,  $t \notin \Omega$  can be recovered if the Fourier transform of the signal  $f(\omega)$  is sparse in a particular domain. The recovered signal  $g(\omega)$  can be computed by solving the constrained optimization problem

$$\min_g \sum_{\omega} |g(\omega)| \quad \text{s. t. } f(n) = g(n), n \in \Omega. \text{ No training data are needed for this reconstruction}$$

as it is solely based upon the sparsity of the data. If only small parts of the image move at the full temporal bandwidth, the Fourier transform along the temporal dimension ( $x$ - $f$  space) provides a sparse representation (3) of the data and no further sparsifying transform such as the wavelet transform (4) is needed.

## Methods:

Data were acquired on a 1.5T Philips Achieva whole body MR system (Philips Medical Systems, Best, NL) using a 5-element coil array. A balanced SSFP sequence with high temporal resolution for resolving the dynamics of valvular leaflets was used with the following parameters: spatial resolution=1.6 x 1.6 x 8 mm<sup>3</sup>, TR=3.4 ms,  $\alpha$ =60°, cardiac phases=78. The  $k$ -space was randomly undersampled along the phase encoding and temporal dimensions by a factor of 3 resulting in a total scan duration of about 17s allowing for single breath hold acquisitions. With  $k$ - $t$  BLAST, an undersampling factor of 4 was used which corresponds to a net acceleration factor of 3.36 and 2.63 with 11 and 33 training profiles, respectively. The reconstruction of the randomly undersampled data was done offline using MATLAB (The MathWorks, Natick, MA, USA). Although a convex  $L^1$  minimization problem is mathematically tractable it is computationally demanding due to the large number of free variables. Therefore a simple steepest descent algorithm along the energy difference between the reconstructed and the measured  $x$ - $f$  space was used. Due to the properties of the point-spread function of a random sampling scheme this results in a minimum  $L^0$  norm of the  $x$ - $f$  space which features the same properties as the  $L^1$  norm.

## Results:

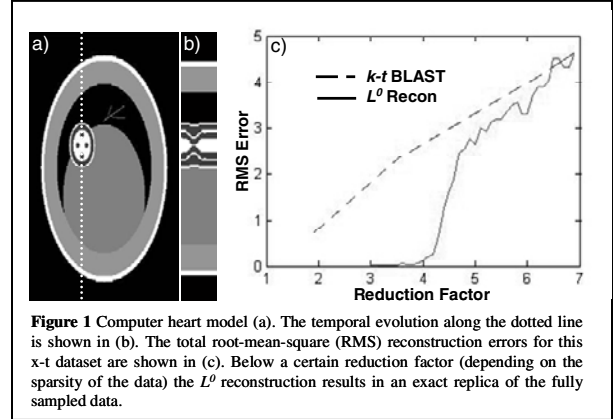
Figure 1 shows simulation results based on a noiseless model data set. It is seen that minimum  $L^0$  norm reconstruction from randomly undersampled data yield an exact reconstruction of the object up to a reduction factor of four while the root-mean-square (RMS) error in  $k$ - $t$  BLAST is significantly larger. Figure 2 compares in-vivo results acquired with  $k$ - $t$  BLAST and random undersampling. As expected, an increased number of training profiles improved the  $k$ - $t$  BLAST reconstruction, yet the  $L^0$  norm reconstruction resulted in better temporal depiction of the valve structure in particular.

## Discussion:

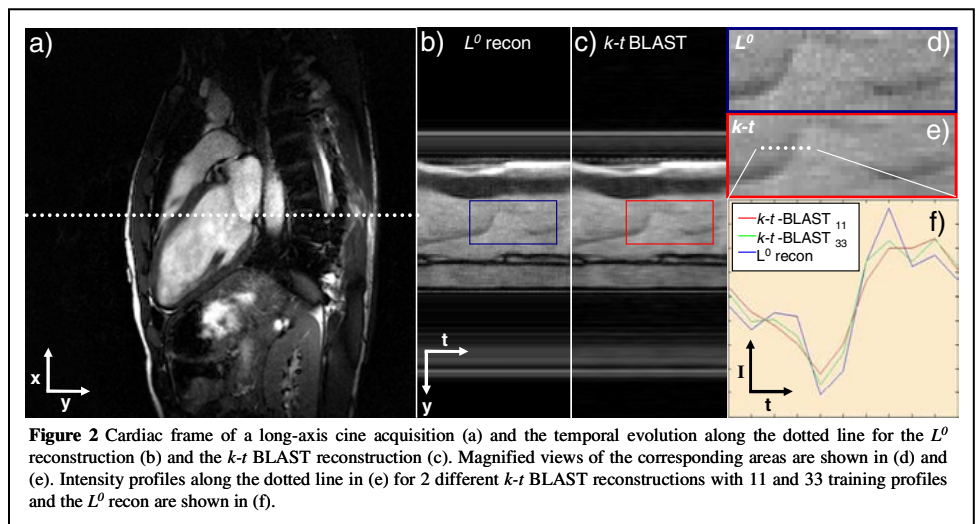
Nonlinear reconstruction methods for undersampled data feature interesting properties. Up to a certain acceleration factor (depending on the sparsity of the data) this reconstruction is exact, resulting in increased image quality and temporal fidelity compared to e.g. the  $k$ - $t$  BLAST reconstruction method. Since the method relies only on the sparsity of the data and not on an additional low resolution training dataset, it can also recover low signal intensities at high temporal frequencies that may get attenuated in the  $k$ - $t$  BLAST reconstruction. The critical acceleration factors below which the reconstruction errors remains negligible depends on the sparsity of the data (and implicitly on the signal-to-noise ratio) as described in (2). Higher acceleration factors result in similar reconstruction RMS errors for both methods although the trait of the artifacts is different. The  $L^0$  reconstruction features incoherent, noise-like artifacts which might be less disturbing to the observer's eye than ghosting or temporal filtering. Acceleration factors beyond the critical factor may be achieved by combining random undersampling with coil encoding principles such as SENSE (5).

## References:

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**Figure 1** Computer heart model (a). The temporal evolution along the dotted line is shown in (b). The total root-mean-square (RMS) reconstruction errors for this  $x$ - $t$  dataset are shown in (c). Below a certain reduction factor (depending on the sparsity of the data) the  $L^0$  reconstruction results in an exact replica of the fully sampled data.



**Figure 2** Cardiac frame of a long-axis cine acquisition (a) and the temporal evolution along the dotted line for the  $L^0$  reconstruction (b) and the  $k$ - $t$  BLAST reconstruction (c). Magnified views of the corresponding areas are shown in (d) and (e). Intensity profiles along the dotted line in (e) for 2 different  $k$ - $t$  BLAST reconstructions with 11 and 33 training profiles and the  $L^0$  recon are shown in (f).