Enforcing strict constraints in multiple-channel RF pulse optimization

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Introduction: Parallel RF excitation has recently been introduced to accelerate multi-dimensionally selective RF pulses in analogy to parallel imaging techniques. Several means of calculating multiple-channel pulses have been presented, e.g. in [1] [2], building upon the analogies between signal reception and low flip angle excitation as well as that between noise in receive signals and power deposition in the transmit mode. The physical equations describing these situations are indeed highly analogous, but the quality criteria and technical constraints are actually quite different. While image noise is basically a minimization target, the specific absorption rate (SAR) and the average and peak powers of RF amplifiers are subject to hard constraints. Closer consideration shows that multiple-channel pulse design differs fundamentally from image reconstruction in that it faces a greater variety of design criteria, of which several need to be strictly enforced. In the present work we address this issue by a novel design approach based on Second Order Cone Programming (SOCP). It permits optimizing the pulse profile under strict limits on global and local specific absorption rate (SAR), as well as on the peak and average forward power of each RF channel.

Method: Let the vector **v** denote the voltage values of a multiple-channel RF pulse discretized in time and stacked for the different channels. Let **m** similarly denote the transverse magnetization resulting from this pulse, written in vector notation after spatial discretization. According to the low flip angle approximation the two vectors are linked by

m=Tv, where the "transmit matrix" **T** reflects the physics of the magnetization during the pulse:

$$T_{\rho(\gamma,\tau)} = B^{+}_{1,\gamma} e^{i\mathbf{k}_\tau \cdot \mathbf{r}_\rho} e^{i\omega(\mathbf{r}_\rho)(dur - t_\tau)} \qquad (1),$$

where ρ,γ, and τ count positions in space ($\mathbf{r}_\rho$), RF channels, and time points, respectively, B_1^+ denotes the transmit RF field, ω(**r**) the local frequency offset, **k** position in excitation k-space, and dur the pulse duration. With respect to the pulse profile **m** the aim of the pulse design is to minimize its deviation from a desired profile $\mathbf{m}_{des}$:

$$\left\| \mathbf{W}\mathbf{T}\mathbf{v} - \mathbf{m}_{des} \right\|_2^2 \rightarrow min \qquad (2)$$

where the matrix W permits spatial weighting, e.g., for masking regions known to be outside the object. The SAR of the pulse in a region of the object can be calculated from the electric field produced by each coil $\mathbf{E}_\gamma$, the conductivity σ(**r**) and density ρ(**r**):

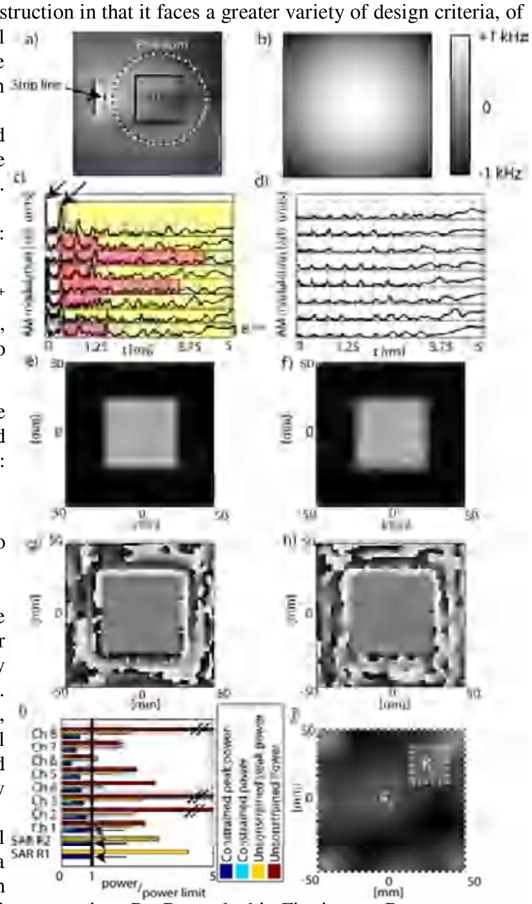
$$SAR_{Region} = \mathbf{v}^H \mathbf{\Psi}_{Region} \mathbf{v}, \qquad \Psi^{(\gamma,\tau)(\gamma',\tau')}_{Region} := \delta_{\tau,\tau'} \int_{Region} \mathbf{E}^*_\gamma(\mathbf{r}) \frac{\sigma(\mathbf{r})}{\rho(\mathbf{r})} \mathbf{E}_{\gamma'}(\mathbf{r}) d\mathbf{r} \qquad (3)$$

The time-averaged forward power $P_{Average}$ and peak power P_{Peak} of each channel are also quadratic in **v**:

$$P_{Average} = \frac{1}{dur} \mathbf{v}_\gamma^H \mathbf{v}_\gamma, \quad P_{Peak} = \max_{(\gamma,\tau)} \left(\left| v_{(\gamma,\tau)} \right|^2 \right) \qquad (4)$$

As opposed to (2) the quantities (3) and (4) cannot be considered as optimization targets but are subject to strict constraints imposed either by patient safety (3) or by the RF system (4). In order to solve this large-scale constrained optimization problem the matrix **T** is iteratively decomposed into a lower rank approximation using a reorthogonalized Lanczos algorithm [3]. For fast multiplication of **T** and $\mathbf{T}^H$ a gridding approach analogous to [4] has been implemented, including iterative off-resonance correction by multi-frequency interpolation. Subsequently all constraints are transformed to the orthogonal basis set defined by the Lanczos vectors calculated in the aforementioned decomposition process and the optimization problem is solved efficiently using SOCP algorithms (e.g. [5], [6]).

To validate pulse design results simulations of the Bloch equations were performed. Eight coil sensitivity patterns (Fig. a) and corresponding electric fields were calculated using FDTD and a realistic model of a single microstrip transmit coil exciting a spherical lossy phantom of 15 cm diameter, having a B_0 off-resonance pattern of up to ±1 kHz (Fig b) . The SAR was calculated for two regions R_1, R_2 marked in Fig. j, were R_1 encompasses the entire object while R_2 represents a local focus, which could in practice correspond to particularly sensitive tissues such as the eyes. The field of excitation (FOX) was set to 0.1 m and the k-space trajectory was a simple 2D, 5 ms, 3-turn spiral with a maximal radius of $k_{max}=160*\pi$ $^{rad}/_{m}$. The target profile was a half-FOX bright square in the centre given on a 64 x 64 matrix, hence the target profile is oversampled by a factor of 4 relative to the resolution of the spiral and the pulse had an acceleration factor of 2.7. The pulse was designed first without constraints and then using SAR restrictions to R_1, R_2 and limiting the peak and average forward power of each channel.



Results: The calculation time in both cases did not exceed 3 min on a standard PC. The resulting pulse waveforms are shown in Figs. c and d. The red / yellow bars show from which time on during the pulse the SAR / total forward power constraints would be exceeded. The spacing of the curves corresponds to the maximum peak power, which is also exceeded, as shown by the arrows in Fig. c. The performance of the pulses are almost equal in profile quality as Figs. e ,f show in transverse magnetisation amplitude and Figs. g, h in phase. Figure i) shows the power used in every channel and deposited SAR in R_1 and R_2 relative to their limits. It can be clearly seen (marked by arrows), that the algorithm finds the pulse at the limit of all constraints except the peak power (because the SAR limits were more restricting) and none of the limits was exceeded.

Conclusions: The proposed SOCP-based approach permits optimizing pulse profiles within strict power and SAR constraints in quite short calculation times. No empirical, problem-dependent regularisation parameter need to be chosen since the algorithm works with constraints imposed directly on physical parameters such as SAR and RF power. The algorithm showed stable performance in all tests and reliable error control within the SOCP solving process.

Figure captions: a) Sensitivity pattern of a single strip line used for the simulation in logarithmic scaling, b) B_0 offresonance pattern, c) RF waveforms resulting from unconstrained pulse design, showing the exceeded power consumption, the red bars start where the SAR limit is exceeded in any region, the yellow bars shows the part of the pulse the amplifier could not display due to its power limits, d) Constrained pulse design RF waveforms without any excess power, e) and f) Simulated magnitude transverse magnetisation produced by waveform showed in c) and d) using the same colour scaling, g) and h) phase of transverse magnetization corresponding to e) and f), i) power consumption of both pulses relative to the power constraints set, the arrows show that the algorithm utilizes the maximum allowed power to achieve best magnetization profile j) SAR distribution within the FOX calculated for the constrained pulse, showing the regions R_1 and R_2 for whom the total SAR has been constrained.

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