Limitations and High Field Potential of Parallel MRI

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This contribution aims at explaining the limitations and potential inherent to parallel imaging (PI), which have recently been studied based on fundamental physical considerations (1-3). Using the principle of reciprocity (4), PI performance is shown to be determined by the electrodynamic fields of the radiofrequency (RF) receiver coil array. This translates into the following important conclusions: (1) PI performance is inherently limited. (2) The limitation softens for high field strength (B_0) and/or large object size (L). Results are presented using terms of wave optics, in particular the RF wave length λ and the relative object size L/λ. The latter is shown to be a key parameter for characterizing the signal-to-noise ratio (SNR) yield that can be achieved in PI.

Principally, the electrodynamic fields of an array of RF receiver coils are governed by the Maxwell equations. For the following, a homogeneous and isotropic object is considered, where coils are exclusively arranged outside the sample. In this case, the Maxwell equations can equivalently be stated in the adimensional Helmholtz representation as (3,5):

$$\left(\Delta+\kappa^2\right)\mathbf{E}\left(\mathbf{r}/L\right)=0,\ \nabla.\mathbf{E}\left(\mathbf{r}/L\right)=0,\ \mathbf{H}\left(\mathbf{r}/L\right)=\frac{1}{i\gamma B_0\mu L}\nabla\times\mathbf{E}\left(\mathbf{r}/L\right) \qquad [1]$$

with $\kappa=\sqrt{\mu\gamma B_0\left(\varepsilon\gamma B_0+i\sigma\right)}L$ the adimensional wave number, $\mathbf{E}$ and $\mathbf{H}$ the electric and magnetic field, respectively, $\mathbf{r}$ the spatial position vector, γ the gyromagnetic ratio, ε the permittivity, σ the conductivity and μ the permeability. Hereby, the characteristic length L (i.e. the diameter of the object) severs as a means to normalize the spatial position vector **r** and to describe the electromagnetic fields on an object size independent, unified length scale. Note that the adimensional wave number in Eq. [1] combines B_0, γ, ε, σ, μ and L to form one complex number, which solely describes the curvature, and therefore the spatial variation, of $\mathbf{E}$ and $\mathbf{H}$. Alternatively, κ can also be expressed in terms of the RF wave length (λ) and the RF skin depth (δ) as (3,5) $\kappa=2\pi L/\lambda+i\,L/\delta$. In this form κ serves as a regime indicator, which allows a rough differentiation between the two fundamental regimes of RF wave behavior; i.e. the near-field RF regime at low B_0 and/or small L ($L/\lambda<1$) and the far-field RF regime at high B_0 and/or large L ($L/\lambda>1$) (1,3).

Based on the principle of reciprocity (4), the signal-to-noise ratio (SNR) of PI (SNR^{PI}) can be readily related to the electromagnetic fields of the RF receiver coil array (1,2). Doing so, two conceptually different loss mechanisms can be identified, with respect to the SNR for full Fourier encoding (SNR^{full}), according to (6):

$$SNR^{PI}=\frac{SNR^{full}}{\sqrt{R}g} \qquad [2]$$

Hereby, the square-root of the reduction factor (R) accounts for SNR loss due to R-fold reduced k-space sampling, whereas the geometry factor accounts for additional noise amplification related to the conditioning of the PI reconstruction. Because the conditioning significantly depends on the local coil sensitivity relations of aliased image pixels, g generally increases with R, is furthermore spatially varying and strongly dependents on the coil-object arrangement. To this end, recently several dedicated PI coil arrays have been designed, aiming at improving both SNR^{full} and g (7-10). Although, partly significant increased SNR efficiency was achieved, these studies also indicate limitations of PI performance, beyond that given by the number of available receiver coils.

In order to investigate the limitations to PI performance on a general basis, the concept of ultimate SNR (11) has been extended toward PI (1,2). The key trick of exploring the fundamental SNR limitations in this way is to not consider a certain, specific RF coil array with associated RF electrodynamic fields $\{\mathbf{E}_C(\mathbf{r}),\ \mathbf{H}_C(\mathbf{r})\}$, but rather to investigate PI performance for a complete set of electric and magnetic Maxwell basis functions $\{\alpha_C(\mathbf{r}),\ \beta_C(\mathbf{r})\}$. Because $\{\alpha_C(\mathbf{r}),\beta_C(\mathbf{r})\}$ covers the entire solution space of Eq. [1] by definition, the corresponding virtual coils can be regarded as the building block for a "complete" coil array.

In the following, ultimate PI performance is described in terms of both SNR^{full} and g [2] assuming a homogeneous spherical object with material properties matched to average in vivo brain conditions (1). For the analysis of the results, the independent variables B_0 and L were summarized to one single parameter in the form of the inverse, relative wave length L/λ, with L chosen to be equal to the diameter of the object (cf. Tab. 1).

B_0 [T]	λ (brain)
1.5T	0.44 m
3.0T	0.27 m
4.7T	0.19 m
7.0T	0.13 m
9.4T	0.1 m
11.5T	0.086 m

Table 1

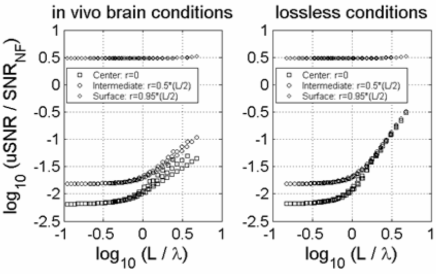

Figure 1: Ultimate SNR^{full} behavior.

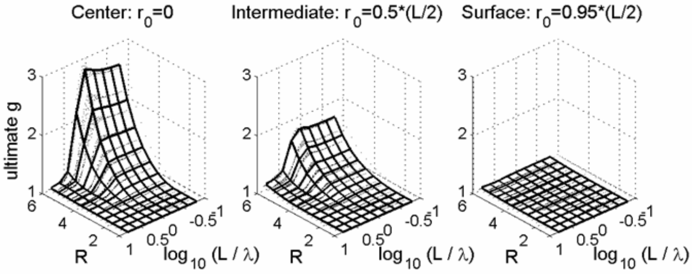

Figure 2: Ultimate geometry factor behavior.

Figure 1 shows the ultimate SNR^{full} (uSNR) normalized by the pure near-field SNR dependency $SNR_{NF} \propto B_0 / \sqrt{L^5 \sigma}$ (cf. Ref. (4) for a derivation), versus L/λ, for three different locations (Center r=0, Intermediate r=0.5*L/2 and Surface r=0.95*L/2) utilizing a double-logarithmic representation. In addition to the results obtained assuming average in vivo brain material properties (left) also the lossless case is shown for comparison (right). In accordance with intuition, uSNR is lowest for the center of the sphere and improves drastically towards the surface. In the near-field regime at low L/λ, the uSNR behavior is to a good approximation governed by SNR_{NF}. However, in the far-field regime at high L/λ, ultimate SNR^{full} starts to increase considerably faster with L/λ than SNR_{NF}. As might be expected, the transition between near-field and far-field uSNR behavior is dependent on the distance between the location of interest and the surface of the sphere (, respectively the location of the virtual, complete receiver coil array). While for the center this transition takes place, when L becomes comparable to λ (i.e. $L/\lambda \equiv 1$), the transition essentially does not happen for locations very close to the surface.

Figure 2 shows the ultimate geometry factor versus R and $\log_{10}(L/\lambda)$ again for three different locations, assuming average in vivo brain material properties. Most notably, the g-factor behavior is important for central locations, where SNR^{full} is typically most critical (cf. Fig. 1). In this region, the ultimate g-factor exhibits two clearly distinguishable operating regimes. While for low reductions g is close to the optimum of 1, for high R values g increases exponentially with R. Hereby, the two wave length regimes again fundamentally impact the appearance of the g-factor behavior. In the near-field regime the critical reduction that separates favorable from unfavorable g-factors, is in between three and four and constant. However, in the far-field regime the critical reduction starts to increase linearly with L/λ. Based on the concept of electrodynamic scaling, this considerable improvement of the g-factor for high B_0 has meanwhile experimentally been confirmed (3). Similarly to the behavior of uSNR, the ultimate g factor improves for locations closer to the surface of the sphere where coil sensitivities are known to be more structured (12).

In conclusion the investigation of ultimate PI performance reveal significant benefits for PI at high B_0 and/or large characteristic length ($L/\lambda > 1$), with respect to both SNR^{full} and g. In addition to general advantages related to the enhanced encoding efficiency of PI (13), this further enhances the specific synergy between PI and ultra-high B_0. Furthermore, the inverse relative wave length L/λ combines multiple dependencies (i.e. B_0, L and the material properties) to form a compact quantity, which characterizes PI performance in a physically meaningful way. In particular, it allows to distinguish near-field RF wave behavior ($L/\lambda < 1$) from far-field RF wave behavior ($L/\lambda > 1$).

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