On the GRAPPA-Operator

Martin Blaimer[1], Felix Breuer[1], Matthias Mueller[1], Robin M. Heidemann[1], Mark A. Griswold[1], Peter M. Jakob[1]

[1]*Experimentelle Physik 5, Physikalisches Institut, Universität Würzburg, Würzburg, Germany*

INTRODUCTION:

The GRAPPA [1] method for parallel imaging is based on an improved AUTO-SMASH [2] reconstruction and uses multiple k-space lines to reconstruct missing lines in every single coil of a multiple receiver coil array. This reconstruction can be reformulated as a matrix operation [3], which allows one to shift signal in k-space, similar to ladder or propagator operators used in quantum mechanics. Furthermore, it can be shown that in principle the reconstruction parameters can be extracted directly from an undersampled data set using the GRAPPA-Operator formalism essentially without any additional information about the coil sensitivities [4].

THEORY AND METHODS:

To shift the signal from an arbitrary position k in k-space to position $k+\Delta k$, an appropriate set of coil weights $\hat{\boldsymbol{G}}$ has to be applied to the signal:

$$\boldsymbol{S}(k+\Delta k) = \hat{\boldsymbol{G}} \cdot \boldsymbol{S}(k) \qquad (1).$$

In this equation, the signal $\boldsymbol{S}(k+\Delta k)$ is a matrix containing the single coil signals S_j of each coil $j=1\ldots N$ of an N element receiver coil array. The set of coil weights $\hat{\boldsymbol{G}}$, which is referred to as the GRAPPA-Operator, is then an N x N matrix. To shift the signal to an other position $k+m\Delta k$, a different set of coil weights has to be applied, which can be expressed by the m-th power of the GRAPPA-Operator:

$$\boldsymbol{S}(k+m\Delta k) = \hat{\boldsymbol{G}}^{m} \cdot \boldsymbol{S}(k) \qquad (2).$$

Therefore, only the GRAPPA-Operator $\hat{\boldsymbol{G}}$ needs to be determined to reconstruct all missing lines of an $m+1$-times undersampled data set.

Furthermore, the GRAPPA-Operator $\hat{\boldsymbol{G}}$ can be directly extracted from a two-times undersampled data set without any additional *a priori* information such as coil sensitivity maps or auto calibration signals. Since $\hat{\boldsymbol{G}}^2$ can be derived from the two-times undersampled k-space, $\hat{\boldsymbol{G}}$ can be obtained by calculating the square root of $\hat{\boldsymbol{G}}^2$. However, the N x N matrix $\hat{\boldsymbol{G}}^2$ has 2^N square roots, but only one square root provides the appropriate coil weights resulting in reconstructions with minimized artifact level. Therefore, it is important to find a method which extracts the appropriate square root. In this abstract, calculation of the image entropy [5] was used to find the best solution.

RESULTS:

Once the GRAPPA-Operator $\hat{\boldsymbol{G}}$ has been determined, all missing k-space lines can be reconstructed. Especially for linear coil arrays, the GRAPPA-Operator method might result in improved reconstruction quality compared to other parallel imaging methods, even at acceleration factors greater than the number of coils ($R>N$).

Figure 1 illustrates the reconstruction of an $R=5$ undersampled simulated data set using only $N=4$ coils of a linear receiver array using normal GRAPPA and the GRAPPA-Operator formalism. For both reconstructions 64 reference lines were used to calculate the reconstruction parameters. While the normal GRAPPA reconstruction results in residual artifacts the image quality of the GRAPPA-Operator reconstruction is improved for the given linear coil array. In this example, normal GRAPPA involves fitting of spatial harmonics of higher order with only 4 coils. Therefore, the lines $\boldsymbol{S}(k+4\Delta k)$ are not accurately reconstructed, resulting in severe image artifacts. In contrast, with the GRAPPA-Operator method only the first spatial harmonic needs to be fitted, which can be done accurately with the underlying coil array.

Figure 2 shows an image reconstruction of an undersampled ($R=2$) data set without any additional *a priori* information, such as coil sensitivity maps or reference k-space lines.

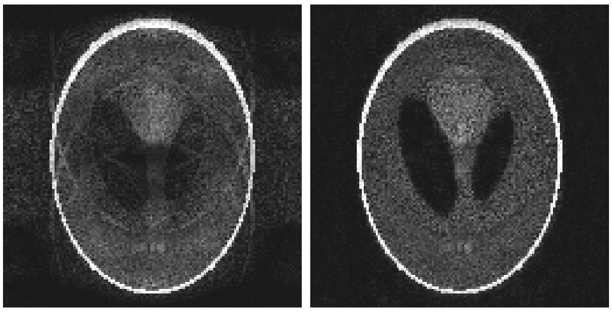

Figure 1: Reconstruction of an $R=5$ undersampled data set with only 4 coils using normal GRAPPA (left side) and the GRAPPA-Operator formalism (right side).

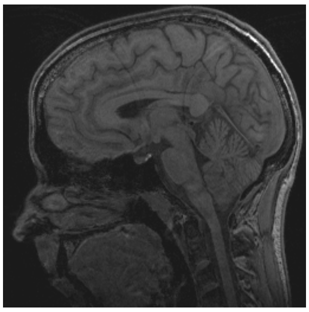

Figure 2: Reconstruction of an $R=2$ undersampled data set without additional *a priori* information using the GRAPPA-Operator formalism and information theory (image entropy).

DISCUSSION:

In this abstract, some interesting properties of the GRAPPA-Operator, a reformulation of the GRAPPA reconstruction, have been presented. Assuming an accurate calculation of the GRAPPA-Operator for an appropriate coil array, this method might result in improved reconstructions at high accelerations. In principle the reconstruction parameters can additionally be determined without additional *a priori* information.

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