Pushing The Speed Limit in SENSE Using Novel Sampling Strategies

Shahin Fattahi[1,2], Brian K. Rutt[1]

[1]Robarts Research Institute, London, Ontario, Canada

[2]Department of Electrical and Computer Engineering, The University of Western Ontario, London, Ontario, Canada

Introduction:

It is well known that the maximum reduction factor achievable by conventional SENSE [1] is limited by the number of coils. Undersampling of k-space at higher reduction factors implies the inversion of an under-determined matrix. Since there is a loss of information due to a very low sampling density, image reconstruction becomes very uncertain in this case, with severe amplification of noise and artifact. Katscher has addressed this issue by using *a priori* information about the image resulting in reduction factors twice as high as the limit in conventional SENSE [2]. In this work, we show that even without any *a priori* knowledge of the image, but rather by exploiting novel sampling strategies and a generalized non-cartesian SENSE reconstruction [3], it is possible to go beyond the conventional acceleration limit in SENSE and still reconstruct a good quality image.

Methods:

In this work, we exploited the widely recognized property of MR images that the majority of the information content is concentrated within a small region at low spatial frequencies whereas the edge information is distributed over a much broader region of higher spatial frequencies. Our fundamental strategy was to undersample the central region with one sampling pattern and reduction factor and the outer region with a different sampling pattern and reduction factor.

A numerical phantom was simulated on an image grid of size 256x256 as shown in Fig. 1(a). The image was then multiplied by simulated coil sensitivities corresponding to an eight channel receiver with the coils distributed azimuthally around the phantom. The synthetic k-space data of each receiver channel was generated by Fourier transforming the corresponding coil image and regridding onto the experimental trajectory. The k-space sampling domain consisted of two circular regions. Region1 was a disk with radius r1 containing mostly the low frequencies and region2 was the remaining part of k-space that extended to a radius of r2 such that region1 and region2 together form a disk of radius r2>r1. The chosen sampling pattern was such that region1 consisted of cartesian points and region2 consisted of points falling on a polar grid as shown in Fig. 1(b). The inner (cartesian) region was sampled with modest reduction factor in an attempt to achieve the minimum noise power, noise correlation and aliasing artifact in the resulting images. The outer region was undersampled by much larger reduction factors, which was primarily achieved through azimuthal undersampling. An important parameter here is the ratio 0<r1/r2<1 (for a fixed r2). We investigated ratios of 0.5, 0.25 and 0.125, in an effort to achieve high net reduction factors without serious loss of critical image features or increase in noise. Through the undersampling strategy described above, net reduction factors of eight and higher (with respect to a fully sampled k-space of $256^2\times\pi/4$ points in the large circular k-space region) were investigated.

Gaussian noise was added to each receiver channel with equal variance in each channel such that an SNR of 25 was achieved in the sum of square of images reconstructed by conventional gridding of the fully sampled purely polar k-space from the corresponding receiver coil [4]. For the reconstruction process we employed the generalized non-cartesian SENSE algorithm (iterative conjugate gradient method) [3]. In the presence of noise, the SENSE encoding matrix becomes ill-conditioned and the iterative method will eventually diverge from the ideal solution in the mean square error sense. However there is always convergence in the first few iterations and the approximate solution reaches a minimum mean square error before noise starts to influence the solution and cause divergence. In the simulations, the iterations were stopped when the approximate solution started to diverge from the ideal image in the mean square error sense [5].

Results:

Figure 2 shows images reconstructed with r1/r2=0.5, region1 at 2x2 acceleration (i.e. 2x reduction in both dimension) and region2 at 40x acceleration along the azimuthal direction (20 radial lines x 65 samples per line) resulting in a total number of 4509 data points per coil and a net reduction factor of 11.4. Approximate solutions at (a) the first iteration (b) the 10th iteration and (c) the same image as (b) without noise, are shown in Fig. 2.

Fig. 3 shows comparative images reconstructed from a purely polar k-space sampling at approximately the same net reduction factor of 11 (4608 data points) with the same amount of noise added as in the case of hybrid cartesian-polar sampling. Approximate solutions at (a) the first iteration (b) the 12th iteration and (c) the same image as (b) without noise.

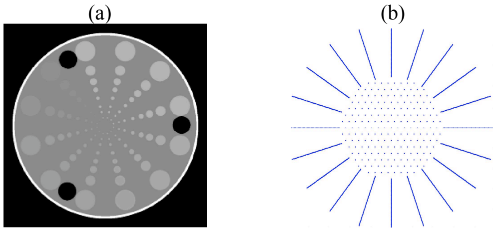
(a) (b)

Fig. 1. (a) simulated numerical phantom; (b) the proposed hybrid cartesian-polar trajectory.

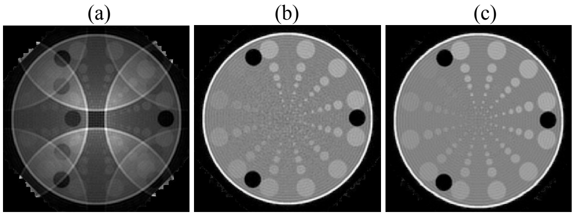
(a) (b) (c)

Fig. 2. Image reconstructed using the hybrid cartesian-polar sampling trajectory at the net reduction factor of 11.4. (a) Noisy image at the first iteration of the CG loop; (b) Noisy image at the 10th iteration of the CG loop; (c) Noise-free image at the 10th iteration of the CG loop.

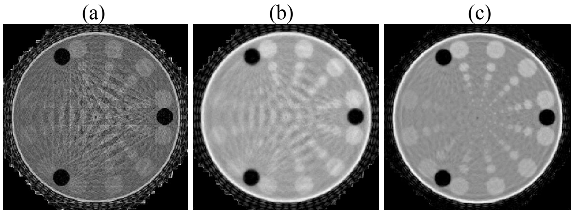
(a) (b) (c)

Fig. 3. Image reconstructed using a conventional polar sampling trajectory at the net reduction factor of 11. (a) Noisy image at the first iteration of the CG loop; (b) Noisy image at the 12th iteration of the CG loop; (c) Noise-free image at the 12th iteration of the CG loop.

Discussion:

These results show that significant image quality improvements are possible using the generalized non-cartesian SENSE reconstruction combined with appropriate choice of non-uniform sampling strategy, compared to the use of more conventional sampling strategies. As our results show, very little penalty is paid in terms of image quality even at very high reduction factors, particularly factors which are beyond the conventionally-stated limit for SENSE. Of course there is always a noise amplification effect as can be seen in the middle of the image in Figure 2. Further simulations at the same reduction factors with r1/r2=0.25 and 0.125 show poorer image qualities which are not shown here due to limited space. Moreover these images show accentuated ring artifacts which could be the result of mismatch of sampling densities at the boundary between the two regions. Further studies are under way based on our hypothesis that with this strategy of hybrid sampling, more image quality or net reduction factor improvements are achievable by using regularization methods as proposed in [2,6].

References:

[1] K. P. Pruessmann et al., MRM 42:952-962 (1999).
[2] U. Katscher et al., Proc. 10th ISMRM 2396 (2002).
[3] K. P. Pruessmann et al., MRM 46:638-651 (2001).
[4] P. B. Roemer et al., MRM 16:192-225 (1990).
[5] E. L. Piccolomini et al., Applied Math and Comp 102:87-99 (1999).
[6] J. Tsao et al., Proc. 11th ISMRM 484 (2003).