

Advanced K-space Techniques

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Introduction

Even though all different parallel imaging techniques essentially solve the same set of linear imaging equations, the various paths taken toward this inverse problem distinguishes the various parallel imaging methods from each other. This talk will focus on k-space based methods which have been developed to deal with specific problems in parallel imaging. Specifically we cover advanced methods to obtain coil sensitivity information with a specific focus on autocalibrating methods. Finally, we will discuss how non-Cartesian (e.g. projection reconstruction and spiral trajectories) impact a parallel imaging reconstruction and how we have solved this basic problem using a segmented GRAPPA approach.

Potential Problems in the Image Domain:

It is clear that whenever accurate coil sensitivity information can be obtained, the SENSE method presented by Pruessmann et al [1] provides nearly optimal results. They have also proposed what has become the standard method to deal with noise in the coil sensitivity maps in the SENSE method. The method is based on a special acquisition designed for coil sensitivity calibration which collects information from both the array and a coil with homogeneous sensitivity.

However, there are many potential difficulties which can prevent acquisition of ideal coil sensitivity maps. Any error which causes the coil sensitivity map to be misaligned with the object can cause artifacts. These errors include simple things such as motion, but could also include truncation artifacts, distortions and chemical shift artifacts resulting from EPI or spiral acquisitions, or even gradient non-linearities in large FOV moving table acquisitions, for example. For these reasons, we began investigation of alternative methods of coil calibration.

Auto-Calibrating Methods

Methods that operate in k-space have different coil mapping requirements, and can therefore be optimized for different imaging situations. The first k-space method SMASH performed the reconstruction in k-space, but actually used coil sensitivity maps in the image domain to determine the reconstruction parameters. For this reason, pure SMASH shares the limitations of the coil mapping technique as in the techniques above. The development of autocalibrated k-space methods (AUTO-SMASH [5], VD-AUTOSMASH [6] and GRAPPA [7]) has removed many of these limitations. In these techniques, a small number of extra lines are acquired before, during or after the acquisition of the undersampled data. The required reconstruction parameters are then determined directly in k-space by fitting one or several lines to other lines in this calibration data set. By fitting data to data, a pure coil sensitivity map is not needed, only the few lines of extra data. No body coil image and no intensity thresholds are needed, thereby generating normal appearing images even in the background. In addition, aliasing in the reconstructed images is not a problem, thereby allowing slightly folded images to be acquired without any problem with the reconstruction. Finally, patient motion is in general not a problem when the extra data is acquired during the acquisition, since these data will accurately track the coil positions as they move. While reconstructions of this type are not guaranteed to be accurate (i.e. free from aliasing artifacts), they can be accurate enough in practice to generate images without any visible artifacts in most cases. In cases where this is not the case, any additional lines acquired in the center of k-space for coil sensitivity mapping can be used in the final image reconstruction to reduce any residual artifacts that may be present, as in refs [6,7].

K-Space Reconstruction as a Convolution Operation

The original k-space method, SMASH, was formulated based on shifting data from one line to the next. In this view, data was always shifted using discrete operations. This general formulation has been the standard up to the original description of the GRAPPA method. While this is still an accurate description of the method, it may be more clear to see this operation as a whole as a convolution operation, as described by Bankson et al [9] or Bydder et al [10].

The original GRAPPA operation was formulated as a discrete shift:

$$S_j(k_x, k_y + m\Delta k_y) = \sum_l \sum_b n(j, l, m, b) S_l(k_x, k_y + bA) \quad [1]$$

However, as implied in [9,10], the entire reconstruction process can be reformulated as a convolution operation on the k-space data:

$$S_j(k_x, k_y + \Delta k_y) = \sum_l \sum_{\tau_x} \sum_{\tau_y} n^l(\tau_x, \tau_y) S_l(\tau_x - k_x, \tau_y - k_y) \quad [2]$$

which is exactly equivalent to Eq. 1 above. This makes more intuitive sense when one considers how the data is distributed in k-space from the beginning. During imaging, the k-space data is convolved with the Fourier transform of the coil sensitivity. This means that information from one line is distributed onto other neighboring lines in k-space. The most natural way to recombine the data for reconstruction of missing data is then also a convolution. In GRAPPA, the weights for this convolution are derived from an extra scan.

Calculation of g-factor maps for GRAPPA

In addition to being more intuitive, Eq. 2 directly implies that this problem could be easily converted into the image domain using the FT. In this case, the reconstructed image is simply the folded image multiplied by the FFT of the weighting functions.

We can use this fact to estimate the noise enhancement for a given GRAPPA reconstruction, without ever using explicit coil sensitivity maps in the reconstruction. The one difference to image domain methods such as SENSE, is that one must first calculate g-factor maps for each individual coil, since each is independently reconstructed. The individual g-maps can then be combined to form an estimate of the total noise enhancement.

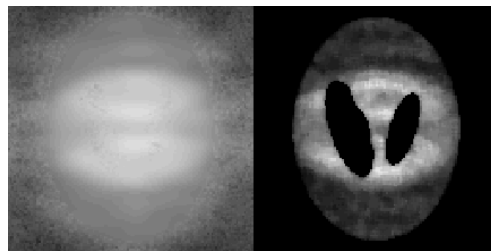


Figure 1. (Left) Approximate g-factor map from a single GRAPPA reconstruction. (Right) G-factor map derived from 60 repeated reconstructions. Note that the areas of increased g-factor are similar in both reconstructions.

The relative noise enhancement for a single coils is simply calculated by the square root of the sum of the magnitude of the FT'd weighting functions for a given pixel. The final composite g-factor map must be combined from the individual g-factor maps using the relative coil weighting functions, such as a sum of square reconstruction for example.

An example is shown for a simulated exam in Fig. 1 for an acceleration factor of 4. Notice that the variations of SNR in the individual maps are similar.

Non-Cartesian Sampling Patterns

All of the methods discussed so far assume the simple aliasing pattern found with normal mode of sampling on a Cartesian grid. The fact that this aliasing is simple allows the simplification of the imaging equations substantially. However, this approach cannot be used with the more complicated aliasing patterns found in non-Cartesian acquisitions, such as projection reconstruction or spiral.

For these trajectories, we must use a different approach using a segmented form of the normal GRAPPA reconstruction. In this method, k-space is broken into many different pieces. Inside of each section, an optimal convolution kernel is found using a fully acquired data set. This kernel is then applied to the undersampled data in that block thereby reconstructing the missing data. This has the primary advantage of being non-iterative, so that the kernel must only be solved for once, which is optimal for dynamic real time studies.

We have applied this general idea to both radial and spiral acquisitions due to their high degree of symmetry. In these cases, significant improvements in reconstruction time was achieved using this segmented GRAPPA approach. Heberlein et al and Heidemann et al will both present results using this general method for spiral acquisitions later in

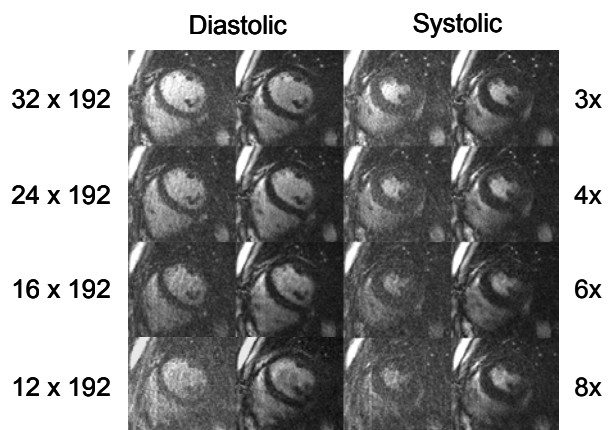


Figure 2: Self-triggered real time radial cardiac imaging with GRAPPA acquired with 32 channel array coil. (1st & 3rd columns) No GRAPPA reconstruction. (2nd & 4th columns) GRAPPA reconstructed images.

the meeting. Examples from the application of the segmented GRAPPA reconstruction for real time free breathing cardiac imaging are shown in Fig. 2 for various levels of acceleration. In all cases, GRAPPA (2nd and 4th columns) results in improved image quality compared to the non-GRAPPA scans (1st and 3rd columns). We see the combination of parallel imaging with these non-Cartesian trajectories as the key to further large increases in imaging speed.

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