On the regularization of SENSE and SPACE-RIP

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INTRODUCTION :

Parallel imaging methods provide accelerated MR image acquisition by reconstructing images from sub-sampled multi-coil *k*-space data. Analytic comparison between different reconstruction methods has been hampered by use of different phase encoding paradigms and regularization approaches, historically unique to each method. Here, we present an analysis that recasts a number of approaches into a common framework to enable an analytical comparison. This analysis shows that when uniform down-sampling along the phase-encode direction is employed, two of these methods (SENSE [1] and SPACE-RIP [2]) can be made analytically equivalent. This enables a clear analytic comparison between the different regularization approaches in SENSE and SPACE-RIP and opens the possibility for hybrid methods, as the results presented demonstrate.

METHODS :

Each of the parallel imaging methods seeks to solve an inverse problem of the form $s = \boldsymbol{P}r$. In the case of uniform down sampling, the normal equations, $\boldsymbol{P}^H s = \boldsymbol{P}^H \boldsymbol{P} r$, become decoupled allowing one to show analytical equivalence between SENSE and SPACE-RIP. The primary difference between them is SENSE solves each decoupled sub-system independently, where-as SPACE-RIP collects all of the sub-blocks for one column, and solves them simultaneously.

In addition, the comparative size of the system matrix formed by each method has led to separate regularization strategies. Specifically, SENSE uses Tikhonov regularization,

$$\min_s \{ \| Pr - s \| + \alpha \| s \|_2 \}$$

where-as SPACE-RIP uses the truncated SVD:

$$r = [\sum_{k=1}^{i} \sigma_k^{-1} . v_k . u_k^H] s$$

In truth, the regularization approach is independent of the parallel MR image reconstruction method. Below, we show that equivalent images can in fact be formed through the use of filter factors to determine appropriate regularization parameter values for each method:

$$\alpha = \sqrt{\sum_{k=i+1}^{n} \sigma_k^2} \quad .$$

RESULTS:

4-coil cardiac data was acquired and reconstructed using water phantom coil-sensitivity estimates. The reconstructed images are shown in Fig. 1, where it can be seen that the anatomical detail is nearly identical. This is particularly true in regions with high coil sensitivity from at least one coil. The dominant difference between the two reconstructions is the manner in which noise in areas of low coil sensitivity is handled. In the truncated SVD reconstruction, Fig. 3(a), signal that corresponds to the modes of the system matrix that have been truncated by regularization appear masked in the reconstructed image.

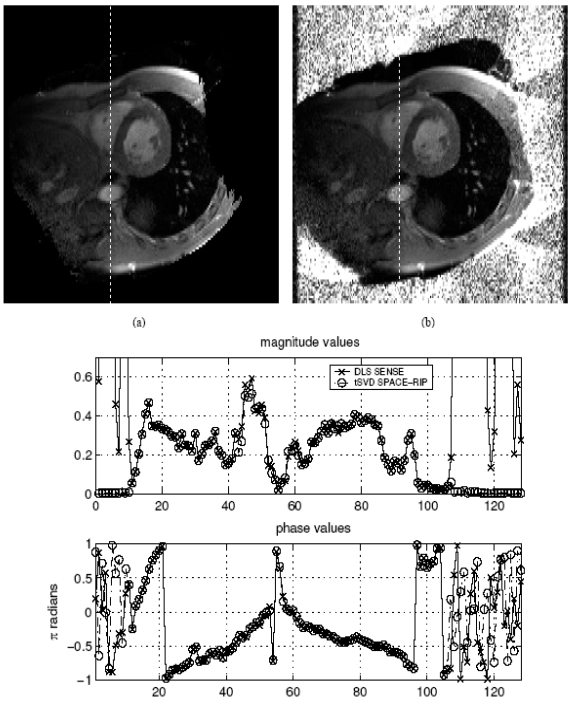

Fig.1: Comparison of (a) tSVD SPACE-RIP and (b) damped-least-squares SENSE reconstructions at identical regularization and windowing levels. (c) shows the complex valued data in the anatomical region for column 101 of each reconstruction, shown by the dotted line in (a) and (b).

DISCUSSION:

Parallel MR imaging is an ill-posed inverse problem. Uniform down sampling decouples the linear system of equations. Using filter factors, the regularization typically used in uniformly down-sampled SENSE and SPACE-RIP can be made equivalent, resulting in nearly identical images. Consequently, this approach suggests how to use truncated SVD regularization in SENSE reconstructions.

REFERENCES:

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