

# Regularization in Parallel Imaging Reconstruction

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## INTRODUCTION

The recent advance of the parallel MRI technology, which utilizes multiple RF receiver array coils [1], has also demonstrated the capability to enhance the spatiotemporal resolution of MRI [2, 3]. In parallel MRI, there exist two major sources in image reconstruction: the first is the reduced data samples in accelerated scans compared to the unaccelerated scans. The second source of noise is the unfolding of the aliased images, which are derived from the reduced sampling and the Nyquist criterion in Fourier imaging. In this study, we focus on the efforts to reduce the noise amplification from this latter cause. We propose to use full field-of-view prior information to condition the encoding matrix, which accounts for the genesis of the observed aliased images in individual RF receivers. The incorporation of prior information is mathematically formulated using the Tikhonov regularization framework. We resort to different approaches to estimate the regularization parameters, including L-curve [4], and SNR-based direct regularization.

The employment of the prior information may decrease the contrast in dynamic scan, while the overall CNR performance has not been investigated. Thus we perform simulations and experiments to study the performance of the regularized parallel image reconstruction in functional MRI experiments. We expect the efforts of optimizing the parallel MRI in brain MRI can be utilize in the investigation of human brain structure and function by improved spatiotemporal resolution and image quality.

## METHOD

In our recent publication [4], we successfully derived the solution of the parallel MRI reconstructions incorporating the prior information using the Tikhonov regularization framework, including the derivation of the associated g-factor metric. We proposed to estimate the regularization parameter using L-curve technique by searching the “elbow” region in the plot of prior error versus model error in the log-log scale [4, 5]. Alternatively, SNR-based direct regularization method is the other approach to estimate the regularization parameter. The SNR of linear equation using whitening observation is then estimated as  $SNR \approx (\tilde{\mathbf{y}}^H \tilde{\mathbf{y}}) / n_c - 1$ , where  $n_c$  is the number of the array channel. Given the SNR estimate, we estimate the regularization parameter from the power spectrum of the singular values of the whitened encoding matrix by searching the singular value with index  $k$  such that following cost function is minimized:

$$\lambda = \underset{s_{ik}}{\arg \min} \left( \left\| SNR - \left( \sum_{i=1}^k s_{ii}^2 \right) / \left( \sum_{i=k+1}^{n_c} s_{ii}^2 \right) \right\|_2 \right)$$

Note that here the regularization parameter is estimated directly without any iterative calculation.

We used the commercial available 8-channel head array coil at 3T (MRI Devices, Waukesha, WI) to validate the reconstruction with/without regularizations in brain structural scans.

To study the CNR of regularized parallel MRI reconstructions in fMRI, we performed simulations based on the 8-channel 3T coil described above by varying BOLD contrast between 1% and 5% and the size of the simulated activated brain region between 36 mm<sup>2</sup> and 576 mm<sup>2</sup>. SENSE acceleration rates were changed between 2.00-fold, 2.67-fold and 4.00-fold accelerations. The averaged true positive rate of detection versus false positive rate of detection without regularization and with L-cure regularization were calculated for each condition separately.

Additional demonstration of the regularized reconstructions was studied using a 23-channel head array coil by calculating the g-factors in 1D and 2D accelerations for our 1.5T scanner.

We used the TSENSE/SHRUG strategies [6, 7] to perform fMRI experiment on 3T scanner with 8-channel head array for block design visual fMRI (4Hz checkerboard). TE was 30 ms. The prior information is derived from the baseline condition. The detection power of SENSE accelerated scans were evaluated based on the images using the composite of all EPI segments. SENSE reconstructed images with the L-curve regularization and without regularization were calculated separately at different acceleration rates.

## RESULTS

Figure 1 shows the benefits of suppressed noise in the regularized reconstructions, which are particularly prominent as the acceleration rate is high (acc: 4.0). While the regularization visually improved quality of reconstruction in high acceleration acquisition, comparing the reconstructions using L-curve regularization or SNR-based direct regularization shows minimal difference discriminated by bare eyes.

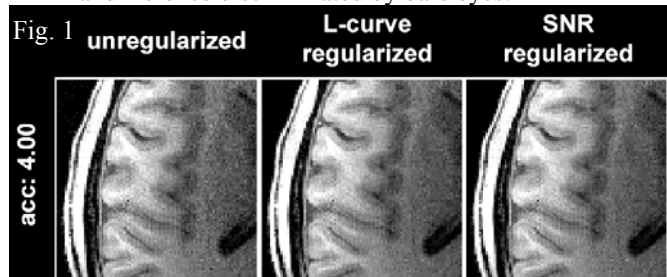


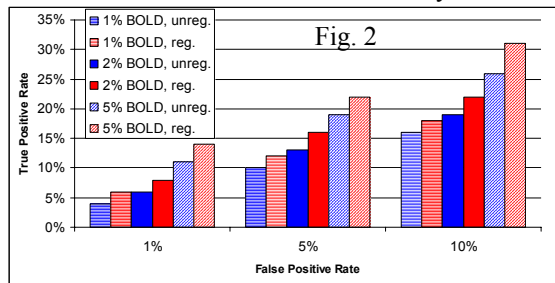
Table. 1 lists the averaged g-factors in unregularized, SNR-based direct regularization and L-curve regularization SENSE reconstructions for 256 X 256 image matrix at 2.00-fold, 2.67-fold and 4.00-fold SENSE accelerations. Note that SNR-based direct regularization is more computationally efficient compared to the L-curve approach. In most cases, the averaged g-factors over the whole image are smaller

when regularization is employed. SNR-based direct regularized reconstructions have smaller g-factors than L-curve regularized reconstructions.

Table 1

SENSE acceleration	unregularized	regularized	
		SNR-reg.	L-curve
2.00	1.081	0.415	0.673
2.67	1.259	1.195	0.781
4.00	1.819	0.521	1.243

The averaged true positive rate of detection versus false positive rate of detection without regularization and with L-curve regularization were plotted in Figure 2 for different BOLD contrast at 4.00-fold acceleration with 144 mm<sup>2</sup> simulated active area. Along with additional simulations, we found that in general the employment of regularization improves the detection at all acceleration rates, BOLD contrasts and sizes of activation brain area in this simulation study.



Images of the individual channel of the customized 23-channel array using proton density weighted FLASH scans to measure SNR and coil coupling were shown in Figure 3, which indicated good localization of coil sensitivity for each individual channel.

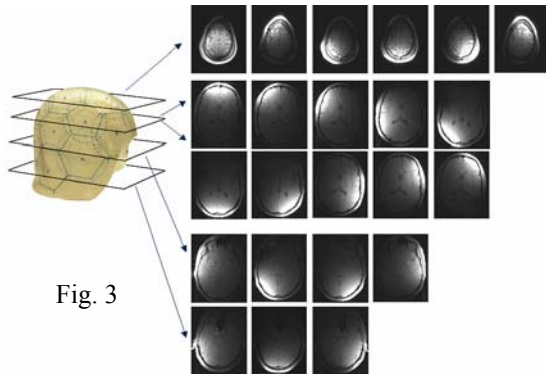


Fig. 3

Simulations on the parallel imaging reconstructions using this 23-channel coil were shown in Table 2, which included both the SENSE acceleration in 1-dimension (1D) and in 2-dimension (2D). The advantage of regularized reconstructions to suppress the unfolding noise was validated by the g-factor maps. Using L-curve regularization, the averaged g-factor was found to be  $1.35 \pm 0.70$ , while unregularized reconstructions with averaged g-factor  $2.00 \pm 0.55$ , in the 16-fold 2D SENSE acceleration.

Table 2	unregularized	regularized
4x acc. in A-P	1.319	1.036
4x acc. in L-R	1.521	1.205
16x acc. in combination (A-P/L-R)	2.007	1.353

Figure 4 shows the *t* statistics maps of 3T visual fMRI experiment using SENSE EPI with/without regularization at 3.0 and 4.0-fold accelerations. At 3.0-fold and 4.0-fold SENSE accelerations, regularized reconstructions yielded larger functional activated area than unregularized reconstructions around the occipital lobe (3X: regularized: 2327 mm<sup>2</sup>; unregularized: 2139 mm<sup>2</sup>; 4X: regularized: 896 mm<sup>2</sup>; unregularized: 735 mm<sup>2</sup>). Figure 5 shows the ROC curves from the *t*-statistical maps using 3.00-fold or 4.00-fold SENSE EPI acquisitions, including both the regularized (solid lines) and the unregularized (dashed lines) unfolding. Using regularization to unfold the identical SENSE acceleration can improve the detection power in both 3.0-fold and 4.0-fold accelerations, as the ROC curves shift toward the upper-left corner.

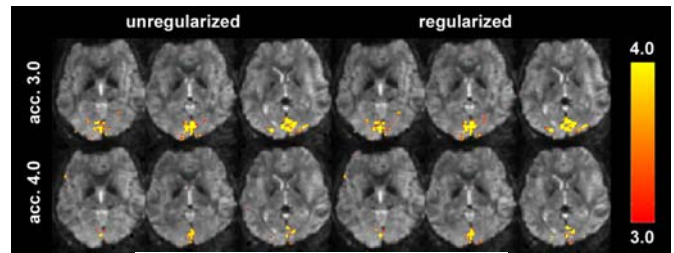
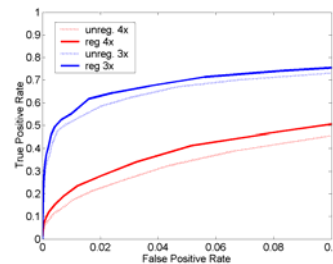


Fig. 5



## CONCLUSION

In this study, we demonstrated the suppress noise amplification in parallel MRI reconstructions using Tikhonov regularization along with L-curve and SNR-based direct estimation of regularization parameter estimation. In both simulations and experiments, we used g-factors and ROC analysis to quantify the advantages of regularization, particularly in functional brain imaging. We expect the efforts of optimizing the parallel MRI in brain MRI can be utilized in the investigation of human brain structure and function by improved spatiotemporal resolution and image quality.

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